# Recent Developments in Dynamic Portfolio Choice

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## Recent Developments

- Multiple risky assets with predictable returns and trading costs: Lynch and Tan (2010) JFQA.
- State-dependent asset returns and labor income growth: Lynch and Tan (2011) JFE.
- State-dependent asset returns, labor income growth, and trading costs:
  - Liquidity premia calculation.
  - Lynch and Tan (2011) JF.

# Multiple Risky Assets, Transaction Costs and Return Predictability: Implications for Portfolio Choice

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# Introduction

- Our analysis considers an economic environment with:
    - transactions costs.
    - return predictability.
    - more than one risky asset.
- Characterize decision-making by a dynamic multi-period CRRA investor.
- Use numerical methods.
- Focus on the two risky asset case.
- Use analysis to examine economic questions:
    - what is the utility cost of redemption fees?
    - what is the utility benefit of being able to borrow?
    - what is the utility benefit of being able to short and how much does this benefit get reduced if the stocks shorted are "special"?
    - a horse race between mutual funds (no shorting but low transactions costs), ETFs (shorting but return reduced by expenses), and individual stocks (shorting but high transactions costs).
    - what is the utility cost of using only one risky portfolio instead of two?
    - what is the utility cost of financial service charges (transaction costs, expenses and redemption fees)?

# Outline

- Relation to Literature
- Motivation
- Limitations
- Decision-making Model
- Portfolio Allocation Rules
- Calibration of Model
- Economic Questions

# Relation to Literature

- Portfolio Choice with Transactions Costs
  - Many Risky Assets: all papers consider a constant opportunity set
    - Liu (2004): CARA agent, analytical for uncorrelated returns and numerical for 10% correlation, assumes form of the solution
    - Leland (2000): agent minimimizes tracking error relative to a target portfolio.
    - Aakian, Menaldi and Sulem (1996): CRRA agent, uncorrelated returns, existence and uniqueness and some numerical results.
  - Single Risky Asset: better understood
    - Constantinides (1986): optimal policy characterized by a no-trade region with return to the closer boundary when rebalancing.
    - Lynch and Balduzzi (2000): predictable returns.
- Portfolio Choice with Margin Requirements
  - Cuoco and Liu (2000): use martingale and duality techniques to establish existence of optimal consumption and investment policies.

# Motivation

- Investors do face transactions costs.
- Investors do trade many assets.
- Returns do seem predictable.

# Limitations

- Numerical results only.

# Decision-making Model

- Wealth Evolution:

$$W_{t+1} = (W_t - C_t)\left(\sum_{i=1}^{N} \alpha_{i,t} R^i_{t+1} + \alpha_{f,t} R^f\right) = W_t(1-\kappa_t) R^W_{t+1}, \tag{1}$$

  and

$$\alpha_{f,t} = 1 - \sum_{i=1}^{N} \alpha_{i,t} - f_t, \tag{2}$$

  - $f$ is the transactions cost per dollar of portfolio value: paid out of the riskless asset.
  - $\alpha$ is a vector of portfolio allocations: unlimited shorting and borrowing.

- Cost Function:

$$f = \phi_{\mathbf{p}}{}'|\alpha - \hat{\alpha}| \tag{3}$$

  - $\phi_p$ is a vector of proportional transactions costs parameters.
  - Can easily incorporate costs that are a fixed fraction of portfolio value.

- Inherited Allocation Evolution:

$$\hat{\alpha}^i_{t+1} \equiv \frac{\alpha^i_t R^i_{t+1}}{R^W_{t+1}} \tag{4}$$

  - where $\hat{\alpha}^i_t$ is the allocation to the $i$th risky asset inherited from time-$(t-1)$.

- Note: $C_t$ is obtained by liquidating costlessly the $i$th risky asset and the riskless asset in the proportions $\hat{\alpha}^i_t$ and $(1 - \hat{\alpha}'_{\mathbf{t}}\mathbf{i_N})$.

- Returns

$$\mathbf{r_{t+1}} = \mathbf{a_r} + \mathbf{b_r} d_t + \mathbf{e_{t+1}} \tag{5}$$

$$d_{t+1} = a_d + b_d d_t + v_{t+1}$$

# Decision-making Model (cont)

- Preferences:

$$E\left[\sum_{t=1}^{T}\delta^t \frac{c_t^{1-\gamma}}{1-\gamma}|D_1,\hat{\alpha}_1\right] \tag{6}$$

  - Time separable and constant relative risk aversion.

- Bellman Equation:

$$\begin{aligned}\frac{a(D_t,\hat{\alpha}_t,t)W_t^{1-\gamma}}{1-\gamma} &= \max_{\kappa_t,\alpha_t}\frac{\kappa_t^{1-\gamma}W_t^{1-\gamma}}{1-\gamma} \\ &\quad +\frac{(1-\kappa_t)^{1-\gamma}W_t^{1-\gamma}}{1-\gamma}E\left[a(D_{t+1},\hat{\alpha}_{t+1},t+1)R_{W,t+1}^{1-\gamma}|D_t,\hat{\alpha}_t\right],\end{aligned}$$

for $t=1,\ldots,T-1,$ (7)

  - Value function homogeneous of degree $(1-\gamma)$ in wealth.
  - Inherited allocations of assets with non-zero transaction costs are state variables.

- Solution Technique:

  - Discretize state and action spaces: use the same 0%, 1%, 2%, ... , 99%, 100% grid for each.
  - Interpolate value function over the state space using its natural triangular tesel-lation: collapses to linear interpolation when only one asset's allocation is a state variable.
  - Develop an algorithm that exploits global concavity of the value function.

# Decision-making Model (cont)

- Extensions:
  - Impose margin requirements on shorting and borrowing.
  - Drive a wedge between the lending and borrowing rates.
  - Allow the rebate rate for shorting to be less than $R^f$.
  - Incorporate a redemption fee for selling an asset within $n$ months of purchase.

# Labor Income Dynamics at Business-cycle Frequencies: Implications for Portfolio Choice

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# Motivation

- **Impact of Hedging Demands Induced by Business-cycle Variation in Labor Income Growth: Not Well Understood**

- **Empirical Literature on Portfolio Holdings of U.S. Consumers**

  **vs**

  **Theoretical Portfolio-choice Literature with Labor Income:**

  **Completely Opposite Conclusions**

  - **Empirical**:
    - *Young: Rich* hold a larger fraction in stock than *Poor*
    - *Young* hold a smaller fraction in stock than *Old*
    - Substantial non-participation, especially for the *Poor*
  - **Theoretical**:
    - *Young: Rich* hold a smaller fraction in stock than *Poor*
    - *Young* hold a larger fraction in stock than *Old*
    - Everyone participates

## Stylized Facts About Individual Labor Income From US Data

- **First two moments of individual income growth move with the business cycle**
  - Intuition suggests **pro-cyclicality in expected income growth**: Point estimates are significant and confirm this (**SDMean**)
  - Storesletten, Telmer and Yaron (STY, 04): strong **counter-cyclicality in volatility of labor income growth** (**SDVol**)
- **Contemporaneous correlation** between labor income growth and stock return: **small positive** (Davis and Willen, 02) (**CorWR**)

## Current Theoretical Literature Fails to Capture these Stylized Facts

- Assumes **shocks to labor income growth and stock return are i.i.d.**
- Further, **correlation** between two is **either artificially high and positive or zero**

## This Paper Captures these Stylized Facts

- Assumes **a joint process for labor income and stock return consistent with stylized facts** and uses **simple VAR** dynamics to switch each of **the three ON** or **OFF**

## Main Results

- **Allowing for business-cycle variation in individual income growth puts theory in line with the stylized facts**
  - Among *Young: Rich* hold a larger fraction in stock than *Poor*
  - *Young* hold a smaller fraction in stock than *Old*
  - Substantial non-participation, especially for the *Poor and Young*

## Main Contributions

- **Recognize the importance of negative hedging demands induced by business cycle variation in labor income growth**
- **Quantify the magnitude of these negative hedging demands**
- **Show these hedging demands bring the model closer to data**
- **Show contemporaneous income growth return correlation has negligible effect when correlation is calibrated to data**

## Illustration Using 20-year Case: Magnitudes of Hedging Demands are Large

- Agent:
    - Has access to a market portfolio of stocks and a riskless T-bill
    - Receives labor income calibrated to a typical wage earner in the “Retail Trade” industry
- Stock return predictability: calibrated to data
- 20 year horizon with monthly rebalancing
- Power utility with relative risk aversion 6

## Quantitative Results for *Young: Rich* vs *Poor*

- Consider *Young* agents with financial wealth to monthly labor income ratios of 1, 10, 1000

| | *Poor* | | *Rich* |
|---|---|---|---|
| | 1 | 10 | 1000 |
| - **Current Literature** (iid income growth uncorrelated with stock return): Higher fractions in stock for *Poor* than *Rich* | | | |
| - All 3 channels switched OFF | 97.5% | 95.5% | 56.9% |
| - **Business-cycle variation** in the 1st 2 moments of income growth as in data: Lower fractions in stock for *Poor* than *Rich* | | | |
| - Add *SDMean* and *SDVol* | 23.2% | 25.8% | 51.5% |
| - **Positive correlation** of income growth and stock return as in data: Almost no effect on allocations | | | |
| - Add *SDMean*, *SDVol* and *CorWR* | 22.0% | 25.2% | 51.4% |
| - Both the first and second moment predictability are contributing to reductions | | | |
| - Add *SDMean* alone | 86.0% | 82.6% | 52.5% |
| - Add *SDVol* alone | 27.2% | 27.3% | 55.6% |

## Quantitative Results for *Young* vs *Old*

- Consider simulation in which *Young* agent's wealth-income ratio is 30

| | *Young* | *Old* |
|---|---|---|
| **Current literature**: *Young* have larger allocation to stock than *Old* | | |
| All 3 channels switched OFF | 91% | 72% |
| **Business-cycle variation** in the 1st 2 moments of income growth as in data: *Young* have smaller allocation to stock than *Old* | | |
| Add *SDMean* and *SDVol* | 38% | 74% |

## Quantitative Results for Non-participation

- Consider *Young* agents with financial wealth to monthly labor income ratios of 1, 10, 1000
- Examine proportion of time a *Young* agent does not participate in the stock market

| | *Poor* | | *Rich* |
|---|---|---|---|
| | 1 | 10 | 1000 |
| **Current Literature**: A *Young* agent participates almost all the time | | | |
| ◦ All 3 channels switched OFF | 2% | 0% | 0% |
| **Business-cycle variation** in the 1st 2 moments of income growth calibrated to data: Substantial *non-participation* especially for the *Poor* | | | |
| ◦ Add *SDMean* and *SDVol* | 73% | 67% | 0% |

## Intuition for Reduction in Stock Holding by *Young* and *Poor*

- **Theoretically**, Merton (1973) for risk aversion greater than 1:
  - Positive correlation between return and future investment opportunities leads to reductions in stock holdings by *Young*
- **Empirically**:
  - Realized stock return is low when the probability of a recession increases
  - But in recessions expected income growth is low and volatility of income growth is high
  - So a *low* stock return this period means increased probability of *low* expected income growth and *high* volatility of income growth in the next period and future periods
  - So stock returns and future "labor income" opportunities are positively correlated
- **Therefore** business-cycle variation in first two moments of income growth:
  - Causes reductions in stock holdings by the *Young*
  - Reductions are more pronounced for the *Poor*, for whom future labor income is more important

## Robustness Checks

- Calibration of the business-cycle variation in permanent labor income volatility:
  - *SDVol* channel's effect on allocations robust to using the NBER variable directly to calibrate the expansion-recession state
- Temporary shocks to labor income: Negligible effect on allocations
- Transitory or Persistent unemployment state: Persistent also reduces stock allocations
- Hump shaped working life profile for earnings and retirement

# Other Related Literature

- **Empirical Literature**: Bertaut (94), Blume and Zeldes (94), Friend and Blume (75), Heaton and Lucas (00), Poterba (93), Vissing-Jorgensen (02), Ameriks and Zeldes (01), Calvet, Campbell and Sodini (06)

- **Theoretical Literature**: Viceira (01), Cocco, Gomes and Maenhaut (05), Davis and Willen (00), Heaton and Lucas (97), Heaton and Lucas (2000), Michaeliedes (03), Benzoni, Collin-Dufresne and Goldstein (06)

- **General Equilibrium** models with countercyclical idiosyncratic income risk

  - Constantinides Duffie (96), Heaton Lucas (96)

- **Other explanations** for low stock holdings: **none a complete explanation** by itself

  - **Participation costs**: Vissing-Jorgensen (02)

  - **Additive internal habit**: Polkovnichenko (03)

  - **Housing**: Cocco (04), Yao and Zhang (02,04)

# Decision-making Model with Labor Income: Exogeneous Processes

## Labor Income

- As in Carroll (96) and (97)

$$y_{t+1} = y^P_{t+1} + \epsilon_{t+1} \tag{1}$$

$$g_{t+1} \equiv \Delta y^P_{t+1} = \bar{g} + b_g d_t + u_{t+1} \tag{2}$$

- $y$ is log labor income, $y^P$ is log permanent income, $\epsilon$ is log temporary labor income
- $d$ is the mean reverting predictor to proxy for the business cycle taken to be the dividend yield
- $\epsilon_t$ and $u_{t+1}$ are uncorrelated i.i.d. processes
- $\sigma_{u_{t+1}}$ can be a function of $d_t$ to allow for heteroskedasticity of permanent income growth

## Returns and Dividend Yield

- As in Campbell and Viceira (99) and Lynch (01)

$$r_{t+1} = a_r + b_r d_t + e_{t+1} \tag{3}$$

$$d_{t+1} = a_d + b_d d_t + w_{t+1} \tag{4}$$

## Decision-making Model with Labor Income: Specifications Considered

- **20-year Specification**:
  - 20 year working life then the terminal date
- **Life-cycle Specification**:
  - 43 year working life followed by 35 years of retirement
  - Mortality rates calibrated to US data
  - Social Security Benefits: Annuity income throughout retirement
  - Earnings Profile: Hump-shaped

# Decision-making Model with Labor Income: Recursive Formulation, 20-year Specification

- Financial Wealth Evolution:

$$W_{t+1} = (W_t + Y_t - C_t)\left[\alpha_t(R_{t+1} - R^f) + R^f\right]$$
$$\text{for } t = 1, \ldots, T-1 \tag{5}$$

- Terminal Condition: $C_T = W_T + Y_T$

- Objective Function:

$$\max_{\{C_t,\ \alpha_t\ \in\ \mathcal{F}_t\}_{t=1}^{T-1}} \ E\left[\sum_{t=1}^{T} \delta^t \frac{C_t^{1-\gamma}}{1-\gamma} | \mathcal{F}_1\right]. \tag{6}$$

- No short selling of stock or borrowing

- Value function:

$$V_t \equiv \frac{a(W_t/Y_t^P, d_t, t)(Y_t^P)^{1-\gamma}}{1-\gamma} \tag{7}$$

  - $W_t/Y_t^P$ and $d_t$ are state variables

- Recursion is solved by iterating backward

# Numerical Solution

- **Difficult features of the problem**:
  - Wealth to period labor income is a continuous state variable with range $[0 \quad \infty]$
  - Value function has high curvature for low wealth/income values
- **Literature**:
  - Heaton and Lucas (97): report that **results can be sensitive to the approximations used**
- **Our paper**:
  - Suggests and implements formal procedures to ensure **results are accurate despite the approximations used**

# Transaction Costs, Return Predictability and Wealth shocks: Implications for Liquidity Premia

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# Introduction and Motivation

- Impact of transactions costs on mean returns.
  - Theory: Constantinides (1986).
    - Liquidity premium:
      - Is the difference in the expected returns on two otherwise identical assets,
      - Only one of which is subject to transactions costs,
      - Such that an investor is indifferent as to which asset she has access to.
    - Standard problem with i.i.d. equity returns and proportional transaction costs: Per-annum liquidity premium is an order of magnitude smaller than the transaction costs rate itself.
  - Empirical:
    - Large differences in mean per-annum risk-adjusted returns of liquidity measure-sorted portfolios: Same order of magnitude as the differential in the transaction cost rate.
    - Brennan and Subrahmanyam (1996): 6.6 % p.a. ab. ret. spread for Kyle's $\lambda$ quintiles.
    - Pástor and Stambaugh (2002): 7.5% p.a. ab. ret. spread for covariance with illiquidity deciles.
    - Others: Easley, Hvindkjaer and O'Hara (2002), Brennan, Chordia and Subrahmanyam (1998), Amihud (2002), Acharya and Pedersen (2002), Hasbrouck (2003).
  - Need to connect and reconcile these seemingly contradictory literatures.
  - One direction is to examine more realistic decision-making models.

# Our Work

- Complicate the standard i.i.d. problem by introducing:
  - Return predictability: calibrated to U.S. data.
  - State dependent wealth shocks:
    - Labor income: Calibrated to U.S. data.
    - Stationary multiplicative.
  - State dependent transactions cost rates: Means calibrated to U.S. data.
- Keep the problem multi-period: Which is important (Constantinides, 1986).
- Allow access to a second high liquidity asset: Important since this access reduces liquidity premia on the low liquidity asset.
- Compare the magnitudes of the implied liquidity premia to that for the standard i.i.d. problem.

# Calibration

- Cost rate calibration: Based on rates estimated for the 5 smallest and the 5 largest size deciles (Lesmond, Trzcinka and Ogden, 1999)
  - Low liquidity portfolio: 3%.
  - High liquidity portfolio: 1%.
- Return calibration: Based on portfolios of NYSE and AMEX stocks formed on basis of Amihud's (2002) illiquidity measure.
  - i.i.d..
  - Predictable using dividend yield.
- Labor income calibration: Based on PSID data for professionals and managers not self-employed under the age of 45.
  - Growth in permanent component allowed to be pro-cyclical as in data.
  - Predictable using dividend yield.

# Main Result

- Add real world complications to the canonical problem; Then per-annum liquidity premia:
  - No longer an order of magnitude smaller than the cost rate.
  - Now can be the same order of magnitude as the cost rate.
- Calibration results:
  - Agent has access to a riskless asset, the value weighted portfolio of the 13 least liquid (Low Liquidity Portfolio) and the 12 most liquid (High Liquidity Portfolio) of 25 liquidity sorted portfolios.
  - Transactions cost rate on the low liquidity portfolio is a constant 2%, the cost spread in the data between the low and high liquidity portfolios.
  - Consider a financial wealth to monthly permanent labor income ratio of 10 in labor income cases.
  - Liquidity premia:
    - 0.08 % p.a. with i.i.d. returns and no-labor income growth.
    - 0.94 % p.a. with i.i.d. returns and i.i.d. labor income growth.
    - 1.12 % p.a. with predictable returns and procyclical labor income growth (Base Case).
  - Premia numbers are close to the Fama-French abnormal return differential between the low and high liquidity portfolios of 1.88 % p.a..

# Comparison with the Standard I.i.d. Problem

- Constantinides (86)
  - For the same agent facing no transaction costs: Has the same optimal allocation every period.
  - Shocks to portfolio compositions due to: Realized returns on component assets.
  - Transactions cost rate: Constant.
- Our Set-up
  - For the same agent facing no transaction costs:
    - Return predictability means the optimal allocation changes with the state each period.
    - Hedging demands mean that the optimal allocation changes with age holding the state fixed.
  - Shocks to portfolio compositions due to:
    - Realized returns on component assets.
    - Realized cash wealth shocks.
  - Transactions cost rate:
    - Unconditional mean is calibrated to data as in Constantinides.
    - But the rate can be higher in states in which transaction costs hurt more.

# Interpreting the Liquidity Premium for a Given Agent

- Three possibilities for a given agent:
  - Marginal agent (holds the asset and sets the price): Agent liquidity premium is the equilibrium liquidity premium.
  - Inframarginal agent(holds the asset but is not marginal): Agent liquidity premium is a lower bound on the equilibrium liquidity premium.
  - Agent does not hold the asset: Agent liquidity premium says nothing about the equilibrium liquidity premium.
- Want to choose an agent whose likely to be holding the asset in question.

# Interpreting the Base Case Liquidity Premia Results

- Base-case agent allocates more to the low than the high liquidity portfolio but in the U.S., market cap is lower for the low than the high liquidity portfolio.
  - So base-case agent appears to be an inframarginal agent: Her liquidity premium likely represents a lower bound on the equilibrium liquidity premium.
  - Heterogeneity across agents must be important for all assets to be held, consistent with market clearing.
  - Delegated portfolio managers can be shown to tilt their portfolios towards the benchmark assets used to determine fund fees (Cuoco and Kaniel, 2006).
    - Benchmark assets are typically liquid.
    - So funds managed on behalf of others hold large fractions of their portfolios in liquid assets.
    - Delegated portfolio managers also help markets to clear with all assets being held.
- Idiosyncratic variation in permanent income constitutes almost all permanent income variation.
  - Trades motivated by changes in human capital value are largely unsynchronized.
  - Net trades can be zero, consistent with market clearing.

# Limitations

- Partial equilibrium.
- Says nothing about how transactions costs affect market clearing prices via their effect on risk-sharing.
- First pass towards an understanding of how equilibrium prices and returns might be affected.

# Relation to Literature

- Seminal work by Constantinides (1986): Calibration result.
- Other work:
  - Heaton and Lucas (1996): Two classes of agent face idiosyncratic labor income risk and transaction costs to trade the risky and riskless assets.
  - Vayanos (1998): Agents trade for life-cycle purposes and all trading is predetermined.
  - Aiyagari and Gertler (1991): Economy has idiosyncratic shocks but no aggregate uncertainty.
  - Huang (2002): i.i.d. liquidity shocks lead to higher premia for riskless assets.
  - Acharya and Pedersen (2002): All pairwise covariances of asset return and liquidity and market return and liquidity can affect expected asset return.
  - Amihud and Mendelson (1986): Single period model.

# Decision-making Model with Labor Income

- Preferences: Time separable and constant relative risk aversion.

$$E_t\left[\sum_{t=1}^{T}\delta^t\frac{c_t^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

- Labor income process (follow Carroll 1996 and 1997):

$$y_t = y_t^P + \epsilon_t, \tag{2}$$

$$g_t \equiv y_t^P - y_{t-1}^P = \bar{g} + b_g d_t + u_{t+1}. \tag{3}$$

  - $y_t$ is log labor income, $y_t^P$ is log permanent labor income and $d_t \equiv \ln(D_t)$.
  - $\epsilon_{t+1}$ is log temporary labor income, and $\epsilon_t$ and $u_{t+1}$ are uncorrelated i.i.d. processes.

- Wealth evolution:

$$W_{t+1} = (W_t + Y_t - c_t)(1-f_t)\left[\alpha_{\mathbf{t}}'(\mathbf{R_{t+1}} - R_t^f\mathbf{i_N}) + R_t^f\right]$$
$$\text{for } t = 1, \ldots, T-1, \tag{4}$$

  - $Y_t$ is labor income received at time $t$.
  - dollar transaction costs are $(W_t + Y_t - c_t)f_t$.

## Decision-making Model with Labor Income (cont)

- Cost Function: $f_t = \mathbf{\Phi_t}'|\alpha_\mathbf{t} - \frac{\hat{\alpha}_\mathbf{t} W_t}{W_t + Y_t}|$.
  - Labor income is assumed to be received as cash.
  - Allowed to be stochastic:

$$log(1 + \Phi_t) = a_\phi + b_\phi d_t + \omega_{\phi,t+1}. \tag{5}$$

- Transaction Costs: Paid at time $t$ by costlessly liquidating $i$th risky and riskless assets in the proportions $\alpha_t^i$ and $(1 - \alpha_\mathbf{t}'\mathbf{i_N})$.
- Consumption at $t$: Liquidate costlessly $i$th risky and riskless assets in the proportions $\hat{\alpha}_t^i$ and $(1-\hat{\alpha}_\mathbf{t}'\mathbf{i_N})$.
- No short selling.

# Decision-making Model with Labor Income (cont)

- Evolution of Portfolio Proportions:

$$\hat{\alpha}^i_{t+1} \equiv \frac{\alpha^i_t R^i_{t+1}}{\alpha_{\mathbf{t}}'(\mathbf{R_{t+1}} - R^f_t \mathbf{i_N}) + R^f_t} \tag{6}$$

- Value function: $\frac{a(\Gamma_t, D_t, \hat{\alpha}_{\mathbf{t}}, t)(Y^P_{t-1})^{1-\gamma}}{1-\gamma}$.
  - Homogeneous in $Y^P_{t-1}$.
  - State variables: $D_t, \hat{\alpha}_{\mathbf{t}}$ and $\Gamma_t \equiv W_t / Y^P_{t-1}$.
- Bellman Equation:

$$\begin{aligned} \frac{a(\Gamma_t, D_t, \hat{\alpha}_{\mathbf{t}}, t)(Y^P_{t-1})^{1-\gamma}}{1-\gamma} &= E\left[\max_{\hat{\kappa}(\Gamma_t, D_t, \hat{\alpha}_{\mathbf{t}}, g_t, \epsilon_t, \Phi_t, t), \alpha(\Gamma_t, D_t, \hat{\alpha}_{\mathbf{t}}, g_t, \epsilon_t, \Phi_t, t)} \left\{ \frac{\hat{\kappa}_t^{1-\gamma}(Y^P_{t-1})^{1-\gamma}}{1-\gamma} \right.\right. \\ &+ \left.\left. \delta \frac{(Y^P_{t-1})^{1-\gamma}}{1-\gamma} E\left[a(\Gamma_{t+1}, D_{t+1}, \hat{\alpha}_{t+1}, t+1)(\exp\{g_t\})^{1-\gamma} | \Gamma_t, D_t, \hat{\alpha}_{\mathbf{t}}, g_t, \epsilon_t, \Phi_t\right] \right\} | \Gamma_t, D_t, \hat{\alpha}_{\mathbf{t}} \right], \\ &\text{for } t = 1, \ldots, T-1, \end{aligned} \tag{7}$$

## Remark on Numerical Implementation of Labor Income Problem

- Novel features:
  - Endogenous discrete state representation of the value function (as suggested by Gourinchas and Parker (2002)).
  - No extrapolation: Exploit convergence to an easily solved problem with no labor income as W/Y goes to infinity.

## Future Directions: Theory

- Cross-sectional variation in labor income processes: Implications for the cross section of portfolio choices.
- Parameter uncertainty and learning:
  - Dynamic portfolio choice without learning in a discrete-time setting: Barberis (2000) JF.
  - Learning in a continuous-time setting: Xia (2001) JF.
- How housing and labor income together affect dynamic portfolio choices:
  - Household choice between a fixed-rate (FRM) and an adjustable-rate (ARM) mortgage: Cocco and Campbell (2003) QJE.
  - Dynamic portfolio choice with housing, labor income and i.i.d. stock returns: Cocco (2005) RFS.

## Future Directions: Data

- Comprehensive Swedish data set:
    - Efficiency of household portfolios in a static setting: Calvet, Campbell, and Sodini (2007) JPE.
    - Time-series dynamics of individual portfolios: Calvet, Campbell, and Sodini (2009) QJE.