CHAPTER 13

New models to address stylized facts about asset prices

A number of extensions to the canonical model with i.i.d. aggregate consumption growth, a representative agent who has time and state separable von Neumann–Morgenstern utility, and no frictions have been considered in the macro-finance literature in an effort to explain the stylized facts outlined in the previous chapter. This chapter will discuss the most important extensions:

- Habit persistence preferences: a representative agent setting in which preferences can be represented using von Neumann–Morgenstern expected utility but utility is no longer time separable.
- Idiosyncratic labor income: the economy does not have a complete market and a representative agent does not exist.
- Epstein–Zin preferences: a representative agent setting in which preferences can no longer be represented using von Neumann–Morgenstern expected utility, but instead can be represented using non-expected utility.
- Long run risk model: a representative agent setting in which both the conditional mean and the conditional volatility of aggregate comsumption growth are slowly mean-reverting.
- Frictions.

13.1. Habit persistence preferences

Three features. There are three important dimensions to habit models:

- Difference vs. ratio habit models.
  Ratio:
  $$\mathrm{E}\left[\sum_{j=0}^{\infty}\delta^j\frac{(C_{t+j}/X_{t+j})^{1-\gamma}}{1-\gamma}\,\middle|\,\mathcal{F}_t\right],$$
  where X_{t+j} is the habit at time $t+j$.
  Difference:
  $$\mathrm{E}\left[\sum_{j=0}^{\infty}\delta^j\frac{(C_{t+j}-X_{t+j})^{1-\gamma}}{1-\gamma}\,\middle|\,\mathcal{F}_t\right].$$
- Internal vs. external habit models.
  External habit models: habit is determined by current and past consumption of other agents. In equilibrium, with homogeneity, current and past consumption of other agents will equal the current and past consumption of the optimizing agent.

Internal habit models: habit is determined by the current and past consumption of the optimizing agent.

- Specification of the habit function:

$$X_{t+j} = f(C_{t+j}, C_{t+j-1}, \ldots).$$

Example. Utility specification: difference.

$$\mathrm{E}\left[\sum_{j=0}^{\infty} \delta^j \frac{(C_{t+j} - aC_{t+j-1})^{1-\gamma}}{1-\gamma} \,\middle|\, \mathcal{F}_t\right]$$

The first-order condition for external habit is

$$\mathrm{E}[\delta(C_{\tau+1} - aC_\tau)^{-\gamma} R^i_{\tau+1} \mid \mathcal{F}_\tau] = (C_\tau - aC_{\tau-1})^{-\gamma},$$

and so

$$\mathrm{E}\left[\delta\left(\frac{C_{\tau+1} - aC_\tau}{C_\tau - aC_{\tau-1}}\right)^{-\gamma} R^i_{\tau+1} \,\middle|\, \mathcal{F}_t\right] = 1.$$

The first-order condition for internal habit is

$$\mathrm{E}\left[\left(\delta(C_{\tau+1} - aC_\tau)^{-\gamma} - a\delta^2(C_{\tau+2} - aC_{\tau+1})^{-\gamma}\right) R^i_{\tau+1} \,\middle|\, \mathcal{F}_\tau\right]$$
$$= \mathrm{E}\left[((C_\tau - aC_{\tau-1})^{-\gamma} - a\delta(C_{\tau+1} - aC_\tau)^{-\gamma}) \,\middle|\, \mathcal{F}_\tau\right].$$

Notice that for external habit,

$$RRA = -\frac{Cu_{CC}}{u_C} = \frac{\gamma C(C - aC_{-1})^{-\gamma-1}}{(C - aC_{-1})^{-\gamma}} = \gamma \frac{C}{C - aC_{-1}} > \gamma,$$

so habit can increase risk aversion, but

$$R^f_{t+1} = \frac{1}{\mathrm{E}\left[\delta\left(\frac{C_{t+1} - aC_t}{C_t - aC_{t-1}}\right)^{-\gamma} \,\middle|\, \mathcal{F}_t\right]},$$

which is too volatile when γ is set large enough to deliver the equity premium puzzle.

Campbell–Cochrane habit model. This is an external difference model. Define

$$S_t = \frac{C_t - X_t}{C_t},$$

so the first-order condition is given by

$$\mathrm{E}\left[\delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma} R^i_{t+1} \,\middle|\, \mathcal{F}_t\right] = 1.$$

Assume $c_t = \ln(C_t)$ follows

$$\Delta c_{t+1} = g + \nu_{t+1},$$

where $\nu_{t+1} \sim N(0, \sigma_\nu^2)$, and specify an AR(1) model for $s_{t+1} = \ln(S_{t+1})$:

$$s_{t+1} = \min(s_{max}, (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)\nu_{t+1}). \tag{75}$$

So X_t is a complex nonlinear function of current and past consumption. The advantage of this specification is that consumption never falls below habit. Campbell and Cochrane show that at $s_{t+1} = \bar{s}$,

$$x_{t+1} = \ln X_{t+1} = \left[h + \frac{g}{1-\phi}\right] + (1-\phi)\sum_{j=0}^{\infty} \phi^j c_{t-j},$$

where h is the steady-state value of $x_t - c_t$.

The risk free rate is given by

$$\begin{aligned} r^f_{t+1} &= \ln R^f_{t+1} \\ &= -\ln[\mathrm{E}[\exp\{\ln\delta - \gamma(s_{t+1} - s_t + \Delta c_{t+1})\} \mid \mathcal{F}_t]] \\ &= -\ln\delta + -\ln[\mathrm{E}[\exp\{-\gamma((1-\phi)(\bar{s} - s_t) + g) - \gamma(1+\lambda(s_t))\nu_{t+1}\} \mid \mathcal{F}_t]] \\ &= -\ln\delta + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \tfrac{1}{2}\gamma^2(1+\lambda(s_t))^2\sigma_\nu^2. \end{aligned}$$

The second and third terms capture the effect of the investor's intertemporal saving motive on r^f_{t+1}. Regarding the third term, if s_t is low, marginal utility in period t is higher, but marginal utility in period $t+1$ is lower as s_t reverts to its mean. So the agent wants to borrow, which forces r^f_{t+1} up.
The fourth term captures a precautionary saving motive that stems from the volatility of marginal utility at $t+1$. It motivates the agent to save, which forces r^f_{t+1} down.

Campbell–Cochrane set

$$\lambda(s_t) = \frac{1}{\bar{s}}\sqrt{1 - 2(s_t - \bar{s})} - 1, \qquad s_t \le s_{max},$$

where

$$\bar{s} = \sigma_v\sqrt{\frac{\gamma}{1-\phi}} \qquad \text{and} \qquad s_{max} = \bar{s} + \tfrac{1}{2}(1 - \bar{s}^2),$$

which causes the effects of the third and fourth terms to exactly offset, leaving the risk free rate constant. When s_t is low, this functional form for $\lambda(.)$ causes the precautionary saving motive to increase by just enough to offset the investor's increased desire to borrow because marginal utility at t is higher than at $t+1$. Thus, the model has time varying risk aversion

$$RRA = -\frac{C u_{CC}}{u_C} = \frac{\gamma}{S_t} \ge \gamma,$$

with a constant risk free rate

$$r^f_{t+1} = -\ln\delta + \gamma g - \frac{\gamma}{2}(1-\phi).$$

13.2. Idiosyncratic labor income

If agents in an economy receive idiosyncratic uninsurable labor income, a representative agent may not exist for the economy. So using a representative agent model to price assets can produce expected returns that deviate from those found in the economy. Recall that the equity premium puzzle says that aggregate consumption is too smooth and not sufficiently correlated with equity returns to generate Sharpe ratios for equity as high as seen in US for any reasonable risk aversion coefficient. Perhaps