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Inter-market Arbitrage in Sports Betting

Egon Franck
Erwin Verbeek
Stephan Nüesch

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Abstract

Unlike the existing literature on sports betting, which concentrates on arbitrage within a single market, this paper examines inter-market arbitrage by searching for arbitrage opportunities through combining bets at the bookmaker and the exchange market. Using the posted odds of eight different bookmakers and the corresponding odds traded at a well-known bet exchange for 5,478 football matches played in the top-five European leagues during three seasons, we find (only) ten intra-market arbitrage opportunities. However, we find 1,450 cases in which a combined bet at the bookmaker as well as at the exchange yields a guaranteed positive return. Further analyses reveal that inter-market arbitrage emerges from different levels of informational efficiency between the two markets.

* Corresponding author: University of Zurich, Institute for Strategy and Business Economics, Plattenstrasse 14, 8032 Zurich, email: erwin.verbeek@isu.uzh.ch, phone: +41 44 634 29 61

Introduction

Arbitrage is typically defined as "the simultaneous purchase and sale of the same, or essentially similar, security in two different markets for advantageously different prices" (Sharpe & Alexander, 1990; cited in Shleifer & Vishny, 1997, p.35). The efficient market hypothesis relies to a large extent on the assumption that any arbitrage opportunity is exploited quickly once it appears. Smith, Paton and Vaughan Williams (2005) regard it as "the purest form of weak form inefficiency", when price differences permit riskless arbitrage.

Thus far, all empirical papers that have addressed arbitrage in sports betting have concentrated on arbitrage in a single market setting. The traditional forms of sports betting are either a pari-mutuel totalizator system or a bookmaker system. In pari-mutual betting, risk-free arbitrage is not possible; the payout ratio is not ex ante determined because it depends on the final betting volumes placed at each possible outcome. Arbitrage opportunities can only be identified ex post. Hausch and Ziemba (1990) and Edelman and O'Brian (2004) showed that positive returns can be achieved in pari-mutual betting if bets are spread across different betting pools. In testing cross-track betting in major horse races, Hausch and Ziemba (1990) found that ten combined bets generated an average positive return of 9.2%. However, as mentioned previously, formulating a risk-free arbitrage strategy in pari-mutual betting is impossible because the payout ratios are not known ex ante. In the bookmaker system, risk-free arbitrage is possible because the odds are fixed and publicly announced before the event takes place.[1] Empirical studies that tested arbitrage in bookmaker betting discovered arbitrage opportunities when the number of bookmakers analysed was sufficiently large. Pope and Peel (1989) discovered one occasion with a guaranteed arbitrage return of 2% in a data sample containing the odds of four different bookmakers in English soccer betting during the 1981/82 season. Dixon and Pope (2004) did not find a single arbitrage opportunity in the U.K. fixed-odds soccer betting market from 1993 to

[1] In practice, however, there are some remaining sources of risk that have to be considered as we will discuss later in this article.

1996, using data from only three bookmakers. They argued that the disappearance of arbitrage opportunities could indicate improved and more efficient forecasting by bookmakers. Vlastakis, Dotsis and Markellos (2009), however, disagreed with this conjecture. They found 63 cases of arbitrage in a sample covering 12,841 soccer matches across 26 different countries during the 2002/03 and 2003/04 seasons.

In recent years, a novel betting market has emerged: exchange betting. Online person-to-person betting differs from the traditional betting markets in that bettors can not only buy bets (bet *on* a given outcome), but they can also sell bets (bet *against* a given outcome). In exchange betting, individuals can trade their bets at an online platform such as *betfair.com*. The odds are determined by a continuous double auction process that matches supply and demand.

With the recent advent of betting exchanges, the magnitude of arbitrage opportunities is expected to have increased substantially. First, price differences are likely to be larger across, as opposed to within, markets. Empirical studies (Levitt, 2004; Smith, Paton, & Vaughan Williams, 2009; Franck, Verbeek, & Nüesch, 2009) have shown that bookmakers and bettors do not perform equally well in predicting the outcome of sporting events. This may cause systematic price differences between the bookmaker and the bet exchange market. Second, bet exchange companies charge significantly lower commissions than the bookmakers because they, unlike bookmakers, do not bear any risk. And third, to hedge against the uncertainty of the match outcome placing a bet on all possible outcomes of an event is not necessary; a bettor can simply buy favourable odds from a bookmaker and sell it directly, at a higher price, on the bet exchange platform.

This paper is the first to search for guaranteed arbitrage returns by combining bets on the bookmaker market and the exchange market. We therefore collected the posted odds of eight different bookmakers and matched them with the corresponding odds traded at the *Betfair* platform. *Betfair* is a well-known betting exchange that claims to organise 90% of all exchange-based betting activity worldwide (Croxson & Reade, 2008). The data set covers all soccer matches played in the major leagues of the "Big Five" (England, France, Germany, Italy and Spain) during three seasons (2004/05 to 2006/07), for a total sample size of 5,478 matches. Our analysis reveals ten intra-market arbitrage opportunities in which price differences among book-

makers exceed commissions. Inter-market arbitrage is, however, far more likely. We find 1,450 inter-market arbitrage opportunities that yielded an average return of 1.2%. Thus, for more than one in four matches, an optimal combination of odds from a bookmaker and odds traded at the bet exchange yielded a positive return independent of the game's outcome.

The observation that there are numerous inter-market arbitrage opportunities in sports betting begs the question of which of the two markets can be held responsible. Inter-market arbitrage opportunities can arise from pure noise or from unequal levels of informational efficiency between the two markets. Accordingly, in the second part of this paper, we study the comparable efficiency of the different market settings.

We simulate a betting strategy that buys a single bet at the bookmaker market and at the bet exchange market whenever we identified an inter-market arbitrage opportunity. With this simple betting strategy, increasing the mean return from an average of -7.2% to 12.6% is possible at the bookmaker market, whereas the average return at the bet exchange decreases from -1.8% to -4.6%. Thus, individual bettors can achieve higher but risky returns if they use arbitrage opportunities only to detect favourable bookmaker odds without (costly) hedging at the bet exchange. These summary statistics indicate that the odds offered at the bookmaker market are less efficient than the prices traded on the bet exchange market. Further multivariate tests confirm this finding.

The remainder of the paper is structured as follows. In the next section, the theoretical background for arbitrage opportunities within and between the two market mechanisms is introduced. Next, we present the empirical findings on both intra- and inter-market arbitrage opportunities. Subsequently, we test the extent to which each market contributes to creating the inter-market arbitrage opportunities. The final section offers some concluding remarks.

Theoretical background

A betting market is a simple speculative market, where contracts on some future cash flow are traded. The direction of the cash flow is tied to the outcome of a given event; in our context, it is the outcome of a soccer game. In

fixed-odds betting, the size of the cash flow is determined by the odds.[2] Although in special occasions the odds may change over time, the size of the bettor's claim is tied to the initially taken odd and does not depend on subsequent price changes. There are two distinct market mechanisms for fixed-odds betting: The bookmaker and the bet exchange market.

The bookmaker market is (still) the most popular form of sports gambling. Here, the odds are unilaterally determined by the bookmaker and published a few days before the game starts.[3] Bettors can place their bets at these odds while the bookmaker takes the opposite position. In European sports betting, the odds are typically represented as "decimal odds"; they determine the payout ratio on a winning bet. Thus, for each possible outcome e of a sporting event, the bookmaker i posts his odds $o_{i,e}$. At these odds, the bettor can place a wager that the outcome of the match will actually be e. The bettor's expected return of this bet is

$$E[\pi_{i,e}] = \varphi_e(o_{i,e} - 1) + (1 - \varphi_e)(-1) = \varphi_e o_{i,e} - 1\,, \tag{1}$$

where φ_e is the true probability of the outcome e to occur. $(o_{i,e} - 1)$ corresponds to the bettor's net return if the outcome of the match is e (a winning bet), which is multiplied by the probability of this event to happen. As we set the stake of the bet to unity, (-1) is the bettor's net return in case the match's result is not e (a losing bet) times the probability of this particular event. The inverse of the posted decimal odds $\frac{1}{o_{i,e}}$ can be interpreted as the bookmaker's probability of the underlying match outcome to occur. For a given match, the probabilities of all possible events sum up to greater than one because the bookmaker's margin or "overround" is already included in the

[2] An exception is pari-mutual betting, where the bettor's claim is tied to the volumes placed on each side of the event at the moment the market is closed. Here, the betting volumes on all possible outcomes are aggregated and then distributed to the winners according to their relative stakes. Therefore, the bettor's claim is not ex ante fixed but depends on betting volumes. Pari-mutual betting is still common in horse racing but is becoming less important compared to fixed-odds betting.

[3] The bookmakers actually have the right to adjust odds after the market has opened, but they rarely do so (Forrest, Goddard, and Simmons, 2005).

odds, thus $\sum_e \frac{1}{o_{i,e}} \geq 1$. The average bookmaker's margin in our sample ranges between 10.9% (*B365* and *Gamebookers*) and 14.1% (*Interwetten*).

The person-to-person bet exchange is a different market mechanism. Here, it is not the bookmaker but another bettor taking the opposite side of a contract. Thus, individuals can directly trade the bets with each other at a platform where the bettors post the prices under which they are willing to place a bet - *on* or *against* - a given outcome. The latent demand for wagers is collected and presented in the order book that publicly displays the most attractive odds with the corresponding available volumes. The bettor has the choice to either submit a limit order and wait for another participant to match her bet or to submit a market order and directly match an already offered bet. As a result, there is a continuous double auction process taking place at the platform. If two bettors with opposing opinions agree on a price, their demands are automatically translated into a transaction. Thus, the odds traded at a bet exchange are not determined by a specific market maker, e.g., the bookmaker, but are the result of a continuous matching of supply and demand. The provider of the platform typically charges a commission fee c on the bettors' net profits. The commission fee of *Betfair*, for example, ranges between 2% to 5% percent depending on the individual's annual betting activity.

After a bet on the outcome e of a given event has been matched, both individuals hold a contract on a future cash flow. The size of the cash flow is determined by the agreed odd $o_{ex,e}$, and the direction of the cash flow depends on the actual outcome of the underlying event combined with the position a given bettor holds. He can either hold the "long position" or the "short position". If a bettor goes long, he bets that the outcome e will occur. The expected return in that case is similar to wagering at the bookmaker market with the exception that the commission fee c on his net winnings has to be included, where $0 > c > 1$. Therefore, the bettor's expected return on a *long position bet* with an agreed price $o_{ex,e}$ is

$$E\left[\pi_{ex,e}^{long}\right] = \varphi_e\left(o_{ex,e} - 1\right)(1-c) + (1-\varphi_e)(-1)$$
$$= \varphi_e\left[o_{ex,e}(1-c) + c\right] - 1\,. \tag{2}$$

As already mentioned, Equation (2) is similar to Equation (1) besides the fact that the commission fee c is charged on the winning bet, whereas the bookmaker's margin is already included in the offered odds.
Alternatively, if a bettor goes short, he bets that the outcome e will *not* occur. The expected return on a *short position bet* with $o_{ex,e}$ is

$$E\left[\pi_{ex,e}^{short}\right] = \varphi_e(-1) + (1-\varphi_e)\frac{1}{o_{ex,e}-1}(1-c)$$
$$= \frac{1-c}{o_{ex,e}-1} - \varphi_e\left[\frac{1-c}{o_{ex,e}-1} + 1\right]. \tag{3}$$

As Equation (3) illustrates the opposite market position to Equation (2), the expected return when going short also depends on the actual outcome probability φ_e and the traded odd $o_{ex,e}$ inversely entering the equation. Thus, the return on a short position bet, holding φ_e constant, decreases with $o_{ex,e}$.

Next, we examine the composition of arbitrage opportunities in the betting market. An arbitrage bet requires buying a contract at one price and contemporaneously selling the same or equivalent contract at a higher price. The stakes placed on each side have to be chosen such that the return of the combined bet does not depend on the actual outcome of the game. We denote a combined bet fulfilling this condition a *hedged bet*. An *arbitrage opportunity* arises if the price difference exceeds the transaction costs involved. In order to overcome these transaction costs, the arbitrageur must seek for favourable selling conditions offered by other bookmakers (intra-market arbitrage) or by the betting exchange market (inter-market arbitrage).
First, we consider *intra-market arbitrage*. Since, at the bookmaker market, a bet can exclusively be placed on a given outcome to occur, to "sell" a bet re-

quires going long in each complementary outcome of the event. Thus, the arbitrageur has to wager a proportion s_e of his overall stake on each outcome of the match. To overcome the transaction costs, he has to select the most favourable odds $\bar{o}_e = \max_i(o_{i,e})$ from a set of bookmakers $i = \{1,2,..,I\}$. His expected return of the combined bet is then

$$E[\Pi^{intra}] = \sum_e \varphi_e \bar{o}_e s_e - \sum_e s_e \,. \tag{3}$$

To hedge this bet, the stakes s_e have to be spread over the outcomes in such a way that the probability φ_e in the return equation can be dropped. Therefore, the bettor has to choose his stakes inversely proportional to the odds. This implies that the payoff is constant over all possible outcomes of an event, thus $\bar{o}_e s_e = const. \; \forall \, e$. Since we are interested in the return of this betting strategy, we set the overall stake to unity, thus $\sum_e s_e = 1$. These two conditions are jointly fulfilled if the proportion of a one unit wager placed on each outcome of the match is

$$s_e^* = \frac{1}{\bar{o}_e} \frac{1}{\sum_e \frac{1}{\bar{o}_e}} . \tag{5}$$

The equation for the return of the intra-market hedged bet then reduces to

$$\Pi^{intra} = \frac{1}{\sum_e \frac{1}{\bar{o}_e}} - 1 \,. \tag{6}$$

If the differences of the odds of the involved bookmakers are larger than the associated margins, the intra-market hedged bet yields a positive return. Hence, an intra-market arbitrage opportunity arises if

$$\sum_e \frac{1}{\bar{o}_e} < 1 \tag{7}$$

holds.

Next, we examine *inter-market arbitrage*. Here, we include the possibility of placing wagers at the exchange market to hedge a bet. There are two different

ways to hedge a bet in the inter-market arbitrage case. The arbitrageur can take the long position on a given outcome at the exchange market or bookmaker market and can bet on all contrary outcomes, effectively reselling his contract. We will refer to this method as the *long position inter-market arbitrage* strategy. An alternative is to go short at the exchange market in order to "sell" the contract bought at the bookmaker market. We define this method as the *short position inter-market arbitrage* strategy.

The first strategy is very similar to the intra-market arbitrage case. The arbitrageur bets on all possible outcomes at the most advantageous odds. The only exception is that he seeks favourable odds not only from the bookmaker market but also from the exchange market. Hence, he places his bets on $\bar{\bar{o}}_e = \max\left[\bar{o}_e,\ \left(o_{ex,e}(1-c)+c\right)\right]$. The stakes have to be balanced according to

$$s^*_{long,e} = \frac{1}{\bar{\bar{o}}_e} \frac{1}{\sum_e \frac{1}{\bar{\bar{o}}_e}}, \tag{8}$$

and the return on the *long position inter-market hedged bet* is

$$\Pi^{inter}_{long} = \frac{1}{\sum_e \frac{1}{\bar{\bar{o}}_e}} - 1\,. \tag{9}$$

Thus, a long position inter-market arbitrage opportunity arises if

$$\sum_e \frac{1}{\bar{\bar{o}}_e} < 1 \tag{10}$$

holds.

A more elegant way to realise potential inter-market arbitrage returns is to directly sell a bet at the exchange market. Exchange markets offer the possibility to place a bet not only *on* a certain outcome (a *long position bet*) but also *against* the outcome (a *short position bet*). Thus, the *short position inter-market arbitrage* strategy buys a contract at the bookmaker and sells the same contract at a more favourable price at the bet exchange. The expected return of this arbitrage strategy is

$$E\left[\Pi_{short}^{inter}\right] = s_{short,e}\left[\frac{1-c}{o_{ex,e}-1} - \varphi_e\left(\frac{1-c}{o_{ex,e}-1}+1\right)\right] + s_e[\varphi_e\bar{o}_e - 1]\,. \tag{11}$$

The stakes $s_{s\ ort,e}$ and s_e have to be balanced according to the following two conditions (i)

$$s_{short,e}\left[\frac{1-c}{o_{ex,e}-1}+1\right] = s_e\bar{o}_e = const.\ \forall\, e\,, \tag{12}$$

and (ii)

$$s_e = 1 - s_{short,e} \tag{13}$$

in order to hedge the bet. Therefore, the proportion of a one-unit wager placed at the exchange market is

$$s_{short,e}^{*} = \frac{\bar{o}_e}{\bar{o}_e + \frac{1-c}{o_{ex,e}-1}+1}\,. \tag{14}$$

At the bookmaker market, it is simply

$$s_e^{*} = 1 - s_{short,e}^{*}\,. \tag{15}$$

The expression for the return of the *short position inter-market hedged bet* then reduces to

$$\Pi_{short}^{inter} = \max_e\left(\frac{\bar{o}_e\left(o_{ex,e}-c\right)}{\bar{o}_e\left(o_{ex,e}-1\right)-c+o_{ex,e}} - 1\right). \tag{4}$$

Even though the short position inter-market arbitrage strategy actually allows hedged bets on the event level (in our context a home win, a draw, or an away win), it does not make sense to place more than one hedged bet on the same match. Instead, we assume that the bettor only goes for the most attractive short position hedged bet of a particular match. In doing so, the return of a short position inter-market hedged bet is positive if

$$\bar{o}_e > \frac{(o_{ex,e} - c)}{1 - c} \quad (5)$$

holds for at least one outcome of the match. Hence, a short position inter-market arbitrage opportunity arises if the most attractive odd from the bookmaker market exceeds the corresponding odd from the exchange market after adjustment for the charged commission. Condition (17) is intuitive since the arbitrageur requires high odds from the bookmaker because he bets on the outcome at this market. On the exchange market, he needs low odds because he places a short bet that yields a higher payout ratio the lower the odds.

Arbitrage opportunities in European soccer betting

We examine the betting market on European soccer matches. Our data covers all matches played during three seasons (2004/05 to 2006/07) of the "Big-Five" leagues (English *Premier League*, the French *Ligue 1*, the German *Bundesliga*, the Italian *Serie A* and the Spanish *Primera Division*), resulting in a total sample size of 5,478 games. The "Big-Five" leagues have a market share of over 50% in the European soccer market and are estimated to generate revenues of USD 20.4 billion. Average match attendance varied between 21,800 (in the French *League 1*) and 39,500 (in the German *Bundesliga*) during the 2006/07 season (Jones, 2009).

We collected the odds of eight well-known bookmakers available at *football-data.co.uk*, where odds are recorded on Friday afternoons for weekend games and on Tuesday afternoons for midweek games. The bookmakers include *B365*, *Bwin*, *Gamebookers*, *Interwetten*, *Ladbrokes*, *William Hill*, *Stan James* and *VC Bet*. Furthermore, we matched bookmaker data with corresponding betting exchange prices traded at *betfair.com*. As the odds of *Betfair* may (slightly) change over time, we made sure to collect odds at the same time as when bookmakers announce odds. *Betfair* is considered the most prominent bet exchange platform. With a weekly turnover of more than USD 50 million and over two million registered users, *Betfair* accounts for 90% of all exchange-based betting activity worldwide (Croxson & Reade, 2008). It has been online since 2000 and claims to process five million trades a day. In 2008, *Betfair* generated revenues of USD 450 million (*betfair-corporate.co.uk*).

In the following, we calculate the returns of the hedged bets as outlined in the previous chapter. Table 1 presents our findings.

Table 1
Arbitrage opportunities in European soccer betting

	all	arbitrage opportunities
intra-market:		
return of the hedged bets	-0.051 (0.015)	0.019 (0.022)
observations	5,478	10
percentage		0.18%
long position inter-market:		
return of the hedged bets	-0.018 (0.017)	0.014 (0.018)
observations	5,478	687
percentage		12.54%
short position inter-market:		
return of the hedged bets	-0.006 (0.014)	0.012 (0.014)
observations	5,478	1,431
percentage		26.12%

Notes: The table gives an overview on arbitrage bets in the European soccer betting market. It can be seen that the intra-market strategy (upper block) is less attractive than inter-market arbitrage in terms of number of arbitrage opportunities. The short position arbitrage method (lower block) exhibits thereby even higher potential than the long position inter-market method (middle block).

The first column in Table 1 depicts the average returns of the three different hedged bets for all matches, whereas the second column illustrates the cases with an arbitrage opportunity, defined as hedged bets with a positive return. The upper block illustrates the average returns and the frequency of the intra-market arbitrage bets, where only the odds of the eight different bookmakers are considered to form a hedged bet. The average return of all hedged bookmaker bets is -5.1%, which is considerably higher than the average bookmaker

margin of approximately -12%, but it is still negative. Out of 5,478 matches, we find ten arbitrage opportunities with an average return of 1.9%. This is approximately one arbitrage opportunity out of 500 matches, or 0.2%. The frequency of intra-market arbitrage opportunities in our sample is lower than the percentage of arbitrage opportunities of 0.5% found by Vlastakis et al. (2009). We must note, however, that Vlastakis et al. (2009) included not only soccer games of the "Big-Five" leagues, but also a variety of other European leagues and international tournaments in their sample.

The middle and lower blocks in Table 1 illustrate the returns of hedged bets when considering both bookmakers' odds and odds traded on *Betfair*. With the *long position inter-market* strategy, the average return was -1.8%, and 12.5% of all matches offered an arbitrage opportunity. When adopting the *short position inter-market strategy*, the average return increases to -0.6%, with an arbitrage opportunity identified in 26.1% of all matches. Betting only on arbitrage opportunities yields an average return between 1.2% (*short position inter-market*) and 1.9% (*intra-market).* The arbitrage returns are considerably lower than the average arbitrage return found by Vlastakis et al. (2009) of 21.8%. However, the fact that we find an inter-market arbitrage opportunity for more than one in four matches is remarkable.

Figure 1 depicts the density functions of the returns of the hedged bets when following the three different arbitrage strategies to give a more comprehensive picture of returns on hedged bets.

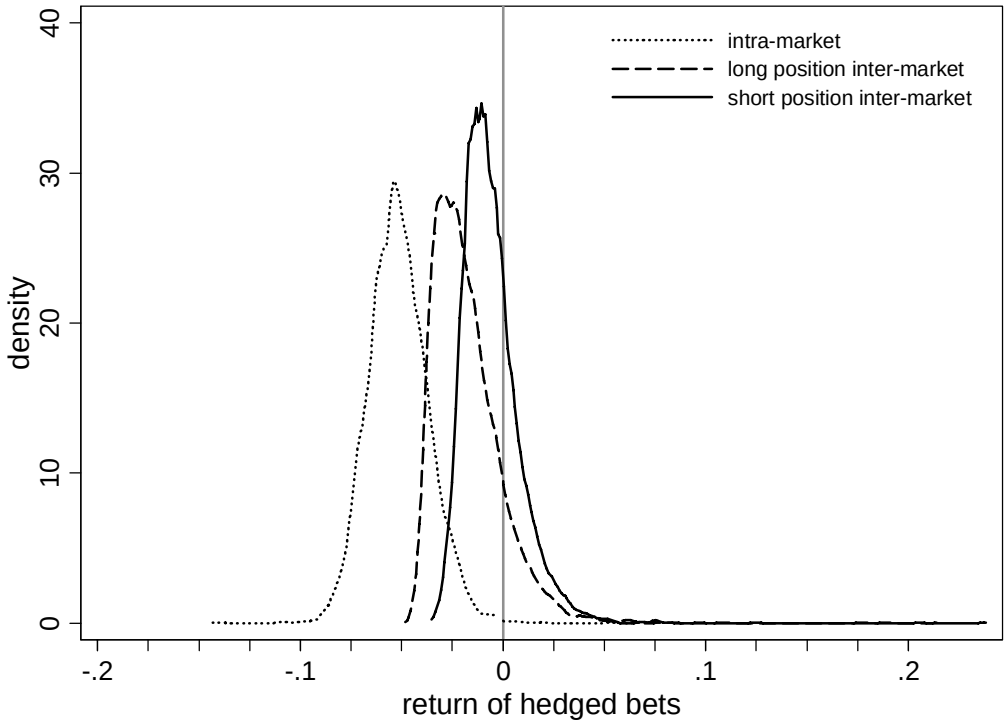

Figure 1: Density functions of the hedged bets' returns

The density functions of the returns from the hedged bets when following the three different arbitrage strategies are depicted.

The *inter-market arbitrage* strategy is far more attractive than the *intra-market arbitrage* strategy. The distributions of returns following one of the inter-market arbitrage strategies are shifted to the right in comparison to the distribution of returns when combining only bookmaker bets. Furthermore, Figure 1 confirms the finding in Table 1 that the *short position inter-market* strategy offers the highest potential for arbitrage. Not only are the returns less negative at the low end of the distribution, but also the area under the curve at the positive side is the largest of all three arbitrage strategies.

As postulated at the beginning of this paper, price heterogeneity is higher across the different market mechanisms than within the same market. The emergence of bet exchange platforms such as *Betfair* largely increased the number of opportunities in which positive returns are available without bearing any risk related to the actual outcome of the game. Inter-market arbitrage opportunities are far more frequent than intra-market arbitrage opportunities.

The short position inter-market strategy is thereby even more profitable than the long position inter-market strategy. Here, the bettor does not have to hedge by going long on the two alternative outcomes, but he can hedge his bet by directly selling it. In doing so, the margins (the bookmaker's overround and the commission of *Betfair*) are charged only twice and not on three bets, as in the long position strategy.

Of course, most matches for which the short position strategy identified an arbitrage opportunity also generate positive returns when following the long position strategy, and vice versa. However, as Table 2 shows, arbitrage opportunities existed exclusively when adopting the short position strategy in 763 cases. In 19 cases, only the long position strategy yielded positive returns.

Table 2
Arbitrage opportunities broken down by method

	intra-market	long position inter-market	short position inter-market
exclusively	2	19	763
with intra-market	-	0	0
with long position inter-market	0	-	660
with short position inter-market	0	660	-
all methods conjointly	8	8	8
total	10	687	1,431
highest arbitrage return	3	164	1,285

Notes: The table shows the number of matches providing arbitrage opportunities broken down by arbitrage method. The first row describes the cases in which only one strategy leads to an arbitrage opportunity. The following rows exihibit the cases where arbitrage opportunities existed by employing different methods. It can be seen that out of 1,452 (1,431+2+19) matches with an arbitrage opportunity, the short position method provides an arbitrage opportunity in 1,431 cases. Furthermore, it permits the highest arbitrage return in 1,285 cases.

Table 2 shows that the short position inter-market arbitrage strategy is by far the most profitable method; it provides an arbitrage opportunity in 1,431 matches, from which it generates the highest return in 1,285 cases. Furthermore, Table 2 illustrates that every arbitrage strategy, for at least some games, offers an arbitrage opportunities exclusively. Since all odds are

known ex ante, it is possible for the arbitrageur to choose for each match the arbitrage strategy with the highest return. When combining all three arbitrage strategies in an optimal way, we find 1,452 arbitrage opportunities. In other words, a guaranteed positive return is possible in 26.5% of all matches.

Arbitrage bets as discussed are riskless in the sense that their return is independent from the outcome of the match. However, there are some remaining sources of risk. First, there is the possibility of a bookmaker running into insolvency and being unable to pay out all winning bets.[4] More relevant than the default risk is the case of cancelled bets. Bookmakers and providers of bet exchange platforms can cancel bets after they have been placed due to, for example, technical problems or suspicion of fraud, and the bettors are returned their initial stakes. If this happens only on one side of the arbitrage bet, then the hedge position is lost, and the return is therefore no longer independent from the actual outcome of the game.
When establishing an arbitrage betting strategy, additional constraints mentioned in the general terms and conditions of bookmakers and bet exchange platforms have to be carefully considered. For example, the arbitrageur cannot place infinitely high wagers as most bookmakers have limits for single bets (typically around USD 10,000 to 15,000). Finally, some online bookmakers (e.g., *Bwin*, *Interwetten*) charge a "handling fee" of 10% when bettors withdraw money from the online account instead of reinvesting it. Thus, not every opportunity found in our sample may guarantee a profitable return in reality, after taking these hidden costs into account.

The comparable efficiency of the two market settings

As demonstrated in the previous section, the European sports betting market as a whole does not satisfy the weak-form efficiency assumption as inter-market arbitrage opportunities are quite frequent. Inter-market arbitrage opportunities may arise from pure noise or from different levels of informational

[4] This could possibly happen if the bookmaker's odds have attracted unequal relative betting volumes on each outcome (resulting in a so-called "unbalanced book") and the actual result of the match stacks against him. In this case, the bookmaker might be unable to pay out winners with the stakes of losers.

efficiency between the two markets. If the former holds true, then price differences between the markets are non-systematically connected to the observed outcome probability and therefore both markets can be seen as equally contributing to the existence of arbitrage opportunities. In the latter case, the price differences, and therefore arbitrage opportunities, are caused by a given market posting odds that are off the efficient level on average, while the other market's predictions tend to be closer to the observed outcome probabilities. Hence, in this section, we study the comparable efficiency of the different market settings.

If a betting market is efficient, then the odds fully reflect all relevant information, such that there is no better predictor of the outcome's true probability than the posted odds themselves. Therefore, no other source of information bears the potential for improving the prediction accuracy of the market's odds. Adapted to our context, the difference in prices between the two betting markets (as an available source of information) does not improve the prediction accuracy of an efficient market.

The expected return of a bet is a function of the true probability and the posted odd. Therefore, the described relationship between true probability and posted odds of an efficient market must translate into observed returns; in an efficient market, the observed returns are not systematically associated with inter-market price differences because the odds already incorporate all relevant information.

We now set out to test this conjecture for both market settings. We concentrate on *short position inter-market arbitrage* opportunities[5] and split up hedged bets into their components; the bet *on* the outcome at the bookmaker market on one side, and the short bet *against* the outcome at the exchange market on the other side. We calculate the bettor's return of the accounts at the different markets from which the hedged bets were derived. According to the efficiency hypothesis, the returns of these accounts should not depend on the existence of an arbitrage opportunity as a signal of inter-market price differences. If both markets are equally efficient, then the price differences that

[5] As outlined in the previous chapter in Table 2, the arbitrage opportunities from the short strategy are by far the most relevant because they account for the highest available arbitrage returns in 88% of all cases (1,285 out of 1,452 matches with an arbitrage opportunity).

enable arbitrage opportunities are unsystematically distributed over the true outcome probability, and both the bookmaker as well as the *Betfair* account should similarly contribute to the observed arbitrage opportunities. If, on the other hand, the odds of the exchange market (bookmaker market) are closer to the efficient level than the odds of the bookmaker market (exchange market), then we expect that bets at the bookmaker market *on* the outcome (the bets at the exchange market *against* the outcome) should perform abnormally well if arbitrage opportunities exist. Table 3 presents the average return of the bookmaker account and the average return of the *Betfair* account whenever we identified a short position inter-market arbitrage opportunity.

Table 3
The composition of short position inter-market arbitrage bets

	arbitrage opportunities
return of arbitrage bets	0.012 (0.014)
- bookmaker account	0.042 (0.477)
- *Betfair* account	-0.030 (0.477)
observations	1,431

Notes: The table summarizes the returns of short position arbitrage bets (first row) and the returns of the accounts from which they are derived; the bet *on* the outcome at the bookmaker market (second row) and the short bet *against* the outcome at the exchange market (third row). It can be seen that the net profit derived from the arbitrage opportunities is accumulated on the bookmaker account, whereas the *Betfair* account serves more as (costly) hedging.

The net profit derived from the *short position inter-market arbitrage* opportunities is accumulated on the bookmaker account, whereas the *Betfair* account serves as (costly) hedging. This finding gives the impression that, on average, the bookmakers' odds are too high in the case of an arbitrage opportunity.

However, the returns of the positions are not directly comparable because the stakes placed at each of the different markets are not equal. In order to hedge against the outcome of the game, the stakes were spread according to Equations (14) and (15). In the following, we drop the idea of optimal hedging and place a one-unit wager at each of the two positions and calculate its ex post return. Again, market efficiency implies that the return of a given position stays at the same level on average, independent of whether the two markets' prices deviate, because price differences that cause an arbitrage opportunity should not reveal any relevant information about the true value of the contracts. Table 4 presents the findings.

Table 4
The returns of the separate positions of short position inter-market arbitrage bets

	all matches all events	arbitrage opportunities
most attractive bookmaker:		
- return of the bookmaker position	-0.072 (1.493)	0.126 (1.489)
- return of the *Betfair* position	-0.018 (0.768)	-0.046 (0.976)
observations	16,434	1,431
random bookmaker:		
- return of the bookmaker position	-0.123 (1.394)	0.146 (1.395)
- return of the *Betfair* position	-0.018 (0.768)	-0.094 (0.938)
observations	16,434	426

Notes: The table summarizes the returns of the positions from which short position hedged bets are derived. This is done for the bookmaker offering the highest odds (first panel) as well as a randomly chosen single bookmaker (second panel). The returns are calculated for all matches and events (first column) and for the cases offering an inter-market arbitrage opportunity (second column). It can be seen that both positions deviate from their average level in the case of an arbitrage opportunity, but the bookmaker position deviates to a larger extent compared to the *Betfair* position.

The first column depicts returns when wagering on all games and on all outcomes and the second column represents cases in which an inter-market arbitrage opportunity existed. Although we selected the bookmaker with the highest odds in the first panel in Table 4, the margins charged are higher here (-7.2%) than at the betting exchange (-1.8%). In order to derive evidence of the comparative efficiency of the two different betting market mechanisms, we compare the average returns of the markets' positions over all events (first column) with the returns of bets for which an arbitrage opportunity existed (second column). The deviation from the average return is more pronounced at the bookmaker market than at the bet exchange. The average return of the bet exchange position decreases from -1.8% to -4.6%, whereas the average return from the bookmaker position improves from -7.2% to 12.6%. Therefore, if bettors place their entire stakes at the bookmaker market whenever inter-market arbitrage was possible instead of hedging against the outcome at the bet exchange, then they could substantially increase their expected profits.

One could argue that this result may be driven by the simple fact that the bookmaker position is composed of the extreme odds of eight different bookmakers and therefore has a higher likelihood to be off the efficient level. In order to control for this issue, we recalculate the average returns if the odds of a random bookmaker are used instead of selecting the most attractive bookmaker odds. The results are presented in the second panel. Our previous findings from the first panel are not sensitive to this specification. Again, when considering a single bookmaker that is randomly chosen, the deviation from the average returns is more pronounced at the bookmaker market than at the bet exchange. If bets are only selected when an arbitrage opportunity exists, then the bookmaker return improves from -12.3% to 14.6%, whereas the return on the bet exchange market changes to a lesser extent, decreasing from -1.8% to -9.4%. Hence, inter-market arbitrage opportunities seemingly serve as a signal of biased bookmaker odds.

Thus far, the differences in prices between the two markets have been reduced to a simple dichotomous signal: whether an arbitrage opportunity exists or not. We next test the robustness of the result when the entire range of price differences is taken into account. We therefore examine how well the odds of a given market predict the outcome of a match and test whether the prediction accuracy could be improved by the additional information about the

price difference between the two markets. To get a market's forecast of an outcome, we transform the odds into implicit probabilities. The implicit probability is simply the inverse of the odd adjusted for a possible margin included in the odds.[6] Thus,

$$p_{ex,e} = \frac{1}{o_{ex,e}} \, 1 \sum_{e} \frac{1}{o_{ex,e}} \tag{18}$$

holds for the implicit probability of the exchange market, and

$$p_{i,e} = \frac{1}{o_{i,e}} \, \frac{1}{\sum_{e} \frac{1}{o_{i,e}}} \tag{19}$$

holds for the implicit probability of the bookmaker market. If the betting market is efficient, then the implicit probability is the best available approximation of the true outcome probability, and the price of a parallel market does not provide any information already present in the market. If the price of the parallel market is able to improve the forecasts of the actual outcome beyond the posted odds, then the efficiency hypothesis is rejected for the betting market at issue. We test the efficiency hypothesis of the two markets by running the following regressions, including the bets of all three outcomes e (win, draw, loss), for each of the 5,478 matches in our sample:

$$P(outcome_e) = G\left(\alpha + \beta_1 \, p_{i,e} + \beta_1 \, \Pi^{inter}_{short\,i,e}\right) \tag{20}$$

$$P(outcome_e) = G\left(\alpha + \beta_1 p_{ex,e} + \beta_1 \, \Pi^{inter}_{short\,i,e}\right). \tag{21}$$

The binary dependent variable $outcome_e$ takes the value 1 if the match's observed outcome was actually the outcome of a given bet; otherwise it takes the value 0. The variable $\Pi^{inter}_{short\,i,e}$ is the return of a short position hedged bet as defined by Equation (16) in the second chapter, except that it is calculated

[6] In the case of the bookmaker, this is the already mentioned overround. The odds traded at *Betfair* can contain an overround as well, because they are determined by members of the betting audience, who trade bets directly with each other. The average bookmaker overround in our sample is 11%, whereas the average overround at *Betfair* is 0.6%.

specifically for a given outcome e and a specific bookmaker i.[7] The return of the short position hedged bet is, as we outlined in the theoretical section, independent from the actual outcome of the game. It is a continuous variable that represents the deviation of prices between the two markets.[8] As we assume that the error term follows a standard normal distribution, we derive our coefficients from a maximum likelihood estimation of a probit model. The marginal effects, the standard errors and the levels of significance are presented in Table 5. The bookmaker is randomly selected from the pool of available bookmakers in each match.

[7] In the theoretical chapter, the return of *a short position hedged bet* was defined as the optimal alternative from the set of all outcomes of a match and the set of all available bookmakers. Following the short position arbitrage strategy, hedged bets are available on the outcome level because the same bet bought at the bookmaker market can be sold at the betting exchange.

[8] Another possibility would be to directly introduce the odds of the two parallel markets into the model. A potential problem of such a procedure is the high multicollinearity between the two variables. Using the return of the hedged bet as the explanatory variable, we circumvent this issue.

Table 5
The different markets' comparable efficiency

	(1)	(2)
implicit probability of the bookmaker	1.189 ***	
	(0.036)	
implicit probability of *Betfair*		1.072 ***
		(0.032)
return of the hedged bet	1.228 ***	0.095
	(0.183)	(0.181)
observations	16,434	16,434
pseudo-R^2	0.088	0.088

Notes: The table presents the additional explanatory power for the outcome of a match, which is rendered by the price differences between the two markets. The dependent variable is the observed outcome of a match. The explaining variables are the probabilities implied by the odds of a randomly chosen bookmaker (first column) or of the exchange market (second column) and the return of the short position hedged bet as a measure of inter-market price difference. The marginal effects of a probit regression (robust standard errors clustered on match level in parantheses) are given. ***, **, * denotes signifcance at the 0.1% , 1%, 5% level respectively. It can be seen that, in contrast to the exchange market, the odds of the bookmaker market do not impound all information rendered by the price differences.

Whereas the return of the hedged bet helps explain the true outcome probabilities at the bookmaker market, it does not do so at the bet exchange. The odds traded at *Betfair* already reflect all relevant information, whereas the bookmaker odds seem biased. At the bookmaker market, the return of the hedged bet renders additional explanatory power beyond the implicit probabilities of the bookmaker odds. The results in Table 5 reveal a significantly positive correlation between the outcome variable and the return of the short position hedged bet return. This implies that whenever the odds posted by the bookmaker are higher than the odds traded at *Betfair* after being adjusted for commission (see Equation 17), the bookmaker odds are on average too high and the implicit probabilities are too low. If the return of the hedged bet is negative because the odds of the bookmaker are lower than the adjusted *Betfair* odds, then the bookmaker odds are on average too low and the implicit probabilities are too high.

Overall, we demonstrated that the price differences between the two markets are not due to noise. Instead, the bet exchange market clearly outperforms the bookmaker market in terms of informational efficiency. Our finding is in line with recent results by Smith et al. (2009) and Franck et al. (2009) showing that the bet exchange odds better predict the actual outcomes than the bookmaker odds.

Concluding remarks

Unlike related literature that concentrates on intra-market arbitrage, this paper investigated inter-market arbitrage in sports betting. Due to the recent emergence of exchange platforms such as *betfair.com*, betting on all possible outcomes of an event in order to hedge against the uncertainty of the match outcome is now unnecessary; a bettor can simply buy favourable odds from a bookmaker and directly sell those wagers at a potentially higher price at the bet exchange market.

We investigated the odds of eight different bookmakers and the corresponding odds traded at *Betfair* on all soccer games played in the top-five European leagues over three seasons (5,478 soccer games). We found (only) ten intra-market arbitrage opportunities, where the most favourable odds across bookmakers were combined. In 1,450 matches, we identified an inter-market arbitrage opportunity. Here, a combined bet at the bookmaker and the exchange market yielded a guaranteed positive return with an average of 1.2%. The likelihood of finding an arbitrage opportunity increased from 0.2% to 26.5% if both bookmaker odds and odds traded at the bet exchange were taken into account.

More detailed analyses revealed that inter-market arbitrage opportunities do not result from random price differences between the two markets; rather, they are due to different levels of information efficiency. We found clear-cut evidence that the bet exchange odds exhibit higher informational efficiency than the bookmaker odds. Bettors could therefore achieve higher expected returns, specifically 12.6%, if they were willing to accept risk and to bet only at the bookmaker market whenever an inter-market arbitrage opportunity was identified.

Our results are striking as they suggest that price anomalies documented so far may be sensitive to the structure of the underlying market. The finding that bookmaker odds are less informationally efficient than bet exchange odds may have different underlying reasons. First, the two markets may face different betting audiences and their prices might simply reflect this instance. Thus, bettors endowed with information and beliefs that are more accurate may place their bets at the exchange market, while less skilled bettors may self-select into the bookmaker market. A hypothetical clean study to test this conjecture would randomly assign either the bookmaker market or the bet exchange market to comparable bettor pools, and then investigate the demand and the resulting prices in both markets. The barriers to performing such experiments in the field are obvious. A second explanation is that both market settings face the same demand structure, but they might not be equally effective with respect to translating dispersed beliefs into efficient prices. Even though there have been initial theoretical attempts (see, e.g., Kuypers, 2000; Levitt, 2004; Franck, Verbeek, and Nüesch, 2007), the literature on gambling markets lacks a comprehensive model of how investor beliefs translate into prices at different market settings. Our results may stimulate further research on this track.

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