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Econometric Issues when Modelling with a Mixture of I(1) and I(0) Variables[#]

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Summary

This paper considers structural models when both I(1) and I(0) variables are present. It is necessary to extend the traditional classification of shocks as permanent and transitory, and we do this by introducing a *mixed* shock. The extra shocks coming from introducing I(0) variables into a system are then classified as either mixed or transitory. Conditions are derived upon the nature of the SVAR in the event that these extra shocks are transitory. We then analyse what happens when there are mixed shocks, finding that it changes a number of ideas that have become established from the co-integration literature. The ideas are illustrated using a well-known SVAR where there are mixed shocks. This SVAR is re-formulated so that the extra shocks coming from the introduction of I(0) variables do not affect relative prices in the long-run and it is found that this has major implications for whether there is a price puzzle. It is also shown how to handle long-run parametric restrictions when some shocks are identified using sign restrictions.

Key Words: Mixed models, transitory shocks, mixed shocks, long-run restrictions, sign restrictions, instrumental variables

JEL Classification: C32, C36, C51

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Econometric Issues when Modelling with a Mixture of I(1) and I(0) Variables

1. Introduction

It seems likely that macroeconometric modelling will involve a mixture of variables that are I(1) and I(0). However most textbooks and applied work deal with the case when all series are I(1), while reviews such as Juselius (2006) make the assumption that all series are either I(1) or I(2). So there appears to be no systematic examination of the estimation issues raised by a mixture of I(1) and I(0) variables.

When there is no co-integration structural models are generally formulated in terms of changes in the I(1) variables. With co-integration present some of the changes in I(1) variables are replaced by error correction (EC) terms when setting up a structural VAR (SVAR). When there are only I(1) variables present in a system, and there is co-integration, traditionally shocks have been classified as permanent and transitory. Now, when I(0) variables are present in the system, they will be in levels, and some assumption needs to be made about the nature of the extra shocks arising from the introduction of these variables into the system. They could either be purely transitory or could have permanent effects on all or some of the I(1) variables. Hence we need to make a distinction between these cases.

When there is a mixture of I(1) and I(0) variables, section 2 suggests a classification of shocks into permanent, transitory and mixed. If the extra shocks have permanent effects then we encounter a difficulty set out in section 2.1. The standard definition of the number of permanent shocks involves the rank of the matrix showing the long-run impact of shocks upon the I(1) variables at infinity - what we will call the long-run impact matrix. As section 2.1 shows, when there are permanent effects of mixed shocks, this definition can no longer apply. It is this fact that gives rise to our terminology of mixed shocks. We keep the term permanent shocks for when there are just I(1) variables and use the "mixed" appellation to account for shocks arising from the introduction of the I(0) variables that have permanent effects.

Section 2.2 then looks at the case where the introduced shocks are transitory. If the extra shocks induced by the presence of I(0) variables are purely transitory then we demonstrate that this requires a particular type of model design. Specifically one needs to force the structural equations to have changes rather than the levels of I(0) variables, which extends result found in Pagan and Pesaran (2008). A good deal of empirical work seems to have this situation in mind but does not seem to recognize that the system needs to be designed to ensure that the shocks are transitory.

Thus studies that have either the growth rate of output or the change in the nominal exchange rate in the SVAR, along with the inflation rate and the level of interest rates, need to design the SVAR to ensure that monetary policy shocks do not have long-run effects on output or relative prices such as the real exchange rate. Canova, Gambetti and Pappa (2007), del Negro and Schorfheide (2004), Smets (1997), and the FAVAR SVAR of Bernanke *et al*. (2005) are examples of papers that have growth rates in the SVAR but do not ensure that monetary policy has zero long-run effects on real variables. We don't think that the researchers working with these systems intended such outcomes but they did not formulate SVAR specifications which ensured that the shocks were transitory.

Section 2.3 turns to an examination of a device that has been suggested as a way of handling mixtures of I(1) and I(0) variables, namely treating the I(0) variables as "co-integrating with themselves". That strategy requires the introduction of "pseudo" co-integrating vectors as well as true ones. Although we use this device in section 2.2 this is done when the extra shocks arising from the I(0) variables are assumed to have purely transitory effects. However, a simple example in section 2.3 shows that this method would provide incorrect answers regarding the permanent components of the I(1) variables if the shocks coming from the introduction of the I(0) variables have permanent effects. Section 2.4 then introduces a general method for computing the permanent component when there are mixed shocks.

It is not possible to study the many papers that feature structures with mixed shocks. Consequently, in section 3 we illustrate some of the outcomes by looking at an influential study by Peersman (2005) which had this feature. Peersman sets up a SVAR involving three I(1) variables and one I(0) variable, with no co-integration between the I(1) variables. Peersman works with three permanent shocks and one mixed shock. The mixed shock is that coming from the introduction of the nominal interest rate. He imposes that interest rate shocks have no effect upon output in the long-run, but they are allowed to affect both the price level and the price of oil in the long-run. As there is nothing imposed on the model to say that these two prices change by the same amount in the long run then the real price of oil must be affected by interest rate shocks at that horizon. This is the same mechanism that makes the real exchange rate respond in the long-run to monetary policy shocks in the application by Smets (1997) mentioned earlier. Examining the implications of this result in Peersman's case we find that the absence of price and output "puzzles" in his estimated model stems from the fact that a monetary policy shock has permanent real oil price effects. When this shock is taken to be transitory the puzzles re-appear.

Now the most common case where mixed shocks arise may be when sign rather than parametric restrictions are applied to identify impulse responses, since these determine only the signs of the responses for a finite number of periods, and nothing is said about the long-run outcomes. Consequently, when SVARs are adopted which include growth rates of output, the monetary policy shocks found from sign restrictions will almost always have a long run impact on the *level* of output. Thus, when Peersman (2005) moved to sign restrictions to identify his monetary policy shocks the resulting impulse responses show long-run effects on real variables. This would be true of many other studies with I(1) variables using sign restrictions to identify shocks. Consequently, in section 4 we look at how one can impose the constraint that mixed shocks are transitory within a sign restrictions framework. Once again we use Peersman's set up to illustrate the approach. Section 5 then concludes.

2. The nature of shocks in structural models with I(0) and I(1) variables

2.1. Definitions of shocks with mixtures of variables

When all variables are I(1) and there is co-integration between them, shocks can be separated into whether they are permanent or transitory. These terms describe the long-run effects on the variables of the shocks i.e. for a shock ε_t and, for the level of a variable y_t, the *long-run* effect will be $\lim_{j\to\infty}\left(\frac{\partial y_{t+j}}{\partial \varepsilon_t}\right)$.[1] Specifically, when a shock is applied that lasts only for a single period it is called transitory if it has a zero effect on all the variables at infinity. A permanent shock is required to have a non-zero long-run effect on *at least one* of the variables. This allows for the possibility that a permanent shock may have a zero long-run effect upon *some* of the I(1) variables. When one adds I(0) variables to this system we will see below that it is necessary to augment the classification. Consequently we will classify the extra shocks induced into the system by the presence of the I(0) variables as being either mixed or transitory.

To appreciate the need for the extended terminology suppose we had a system with three I(1) variables and no cointegration. Then the long-run response matrix $C = \lim_{j\to\infty}\left(\frac{\partial y_{t+j}}{\partial \varepsilon_t}\right)$ will have rows corresponding to the variables y_t and columns representing the shocks. Thus the (1,3) element of *C* is $C_{13} = \lim_{j\to\infty}\left(\frac{\partial y_{1,t+j}}{\partial \varepsilon_{3,t}}\right)$. Now this C matrix might take the form

[1] If y_t is I(1) then it would enter the SVAR as Δy_t.

$$C = \begin{bmatrix} * & * & 0 \\ 0 & * & * \\ * & 0 & * \end{bmatrix},$$

where the * indicate non-zero values. Consequently, as the rank of this matrix is generally three, traditional theory says that there are three permanent shocks.

Now suppose that an I(0) variable is added to the system and that the new shock has permanent effects on all three I(1) variables. This means that the long-run response matrix for the four variables will be[2]

$$C = \begin{bmatrix} * & * & 0 & * \\ 0 & * & * & * \\ * & 0 & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

since the fourth variable is I(0), and so the long-run responses of it to all shocks are zero. Now the rank of this matrix will still be at most three. So, if one used the traditional definition of the rank of the long-run matrix as being the number of permanent shocks, one has the difficulty of describing the nature of the fourth shock. It clearly fails the definition of a transitory shock. Indeed, it looks much like the original three permanent shocks. But, if we call it permanent, then there would be four such shocks and this is in conflict with the rank of the matrix. Therefore, we need to give it a new descriptor, and we will refer to it as a *mixed shock*, since it arises in the context of a mixture of I(0) and I(1) variables. Specifically, *the extra shock associated with the addition of an I(0) variable to a system of I(1) variables is mixed if it has a long-run effect on at least one of the I(1) variables*. If, however, it has a zero long-run effect on all of the I(1) variables, it is transitory, and the long-run response matrix for the four variable case would look like

$$C = \begin{bmatrix} * & * & 0 & 0 \\ 0 & * & * & 0 \\ * & 0 & * & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The rank of this matrix is three and so the rule delivers the correct number of permanent shocks.

[2] Clearly we can differentiate between all shocks here just using the long-run responses. If however the (1,3) element in *C* had been non-zero then we would need some short-run restriction to separate the third and fourth shocks.

We now investigate the implications of including both I(0) and I(1) variables together in systems when the shocks associated with the former are, firstly, transitory (covered in section 2.2) and then mixed (covered in sections 2.3 and 2.4).

2.2. Shocks associated with I(0) variables are transitory

This section shows how to treat I(0) variables in structural models that contain cointegrating relationships among the I(1) variables when the extra shocks coming from the I(0) variables are taken to be transitory.

For simplicity, consider a structural VAR(2) model of n variables of the form

$$A_0 x_t = A_1 x_{t-1} + A_2 x_{t-2} + \varepsilon_t \qquad (1)$$

where A_i are $n \times n$ matrices of unknown coefficients, A_0 is non-singular and ε_t is an $n \times 1$ vector of structural shocks with mean zero and covariance matrix D_n. We assume that there are $n-q$ variables which are I(1) and q which are I(0), while among the I(1) variables there are $p\ (< n-q)$ cointegrating relations. We refer to the latter as the 'true' or 'actual' cointegrating relations as distinct from the q 'pseudo' cointegrating relations coming from the treatment of each of the I(0) variables as 'cointegrating with itself'. This is probably the standard way of handling I(0) variables in SVECMs that is currently in the literature. We will refer to these structures as pseudo-SVECMs.

Because there are p cointegrating relations among the I(1) variables, there are $m = n - q - p$ structural shocks with permanent effects in a pseudo-SVECM. Without loss of generality, let

$$x_t = \begin{pmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{pmatrix},$$

where x_{1t} is the $m \times 1$ vector of I(1) variables whose structural shocks are known to have permanent effects, x_{2t} is the $p \times 1$ vector of I(1) variables whose structural shocks are known to have transitory effects, and x_{3t} is the $q \times 1$ vector of I(0) variables whose structural shocks are transitory by assumption. Let

$$\tilde{\beta} = \begin{pmatrix} \beta_1 & 0 \\ \beta_2 & 0 \\ 0 & I_q \end{pmatrix},$$

where $\tilde{\beta}$ is an $n\times(p+q)$ matrix, and note that there are $p+q$ transitory shocks in the SVECM model. The matrices β_1 and β_2 are $m\times p$ and $p\times p$, respectively. The first column of block matrices in $\tilde{\beta}$ are the coefficients in the 'true' cointegrating relations among the I(1) variables, while the second column gives the 'pseudo' cointegrating relations. The latter involve a coefficient of one on a given stationary variable, and a coefficient of zero on all the remaining variables, and so are represented by the identity matrix. Analogously the loadings vector $\tilde{\alpha}^*$ can be partitioned as

$$\tilde{\alpha}^* = \begin{pmatrix} \alpha_1^* & \delta_1^* \\ \alpha_2^* & \delta_2^* \\ \alpha_3^* & \delta_3^* \end{pmatrix},$$

where $\tilde{\alpha}^*$ is an $n\times(p+q)$ matrix. The sub-matrices α_1^*, α_2^* and α_3^* are of dimension $m\times p$, $p\times p$ and $q\times p$, respectively. Similarly, the sub-matrices δ_1^*, δ_2^* and δ_3^* are of dimension $m\times q$, $p\times q$ and $q\times q$, respectively. The first column of block matrices in $\tilde{\alpha}^*$ shows the loadings on the 'true' cointegrating relations for each group of structural equations while the second shows the loadings on the I(0) variables.

The VAR model of (1) can now be written as the pseudo- SVECM

$$A_0\Delta x_t = -\tilde{\alpha}^*\tilde{\beta}' x_{t-1} + A_2\Delta x_{t-1} + \varepsilon_t . \tag{2}$$

The $p\times 1$ vector of 'true' error correction terms, ξ_t, can be written as

$$\xi_t = \beta_1' x_{1t} + \beta_2' x_{2t} . \tag{3}$$

Following the development in Pagan and Pesaran, we proceed to express the SVECM model of (2) as a structural vector autoregressive (SVAR) model of order two in the variables Δx_{1t}, ξ_t and x_{3t}. From (3), we have

$$\Delta\xi_t = \beta_1'\Delta x_{1t} + \beta_2'\Delta x_{2t} ,$$

from which it follows that

$$\Delta x_{2t} = (\beta_2')^{-1}(\Delta\xi_t - \beta_1'\Delta x_{1t}) , \tag{4}$$

provided the $p \times p$ matrix β_2' is non-singular.[3]

The first m equations in (2) are

$$A_{11}^0 \Delta x_{1t} + A_{12}^0 \Delta x_{2t} + A_{13}^0 \Delta x_{3t} = -\alpha_1^* \xi_{t-1} - \delta_1^* x_{3t-1} + A_{11}^2 \Delta x_{1t-1} + A_{12}^2 \Delta x_{2t-1} + A_{13}^2 \Delta x_{3t-1} + \varepsilon_{1t} , \tag{5}$$

where the A matrices are partitioned conformably with Δx_t. These equations contain the structural shocks with permanent effects. Pagan and Pesaran proved that $\alpha_1^* = 0$ in (5), so that the structural equations with the permanent shocks do not contain the lagged 'true' error correction terms. Here we show additionally that $\delta_1^* = 0$ when the structural shocks associated with the I(0) variables are transitory (have zero long-run effect on all the I(1) variables), i.e. for the SVAR(2) in Δx_{1t}, ξ_t and x_{3t}. Using (4) to eliminate the terms in Δx_{2t} in (5), one obtains

$$\begin{aligned} &(A_{11}^0 - A_{12}^0 (\beta_2')^{-1} \beta_1') \Delta x_{1t} + A_{12}^0 (\beta_2')^{-1} \Delta \xi_t + A_{13}^0 \Delta x_{3t} \\ &= -\alpha_1^* \xi_{t-1} - \delta_1^* x_{3t-1} + (A_{11}^2 - A_{12}^2 (\beta_2')^{-1} \beta_1') \Delta x_{1t-1} + A_{12}^2 (\beta_2')^{-1} \Delta \xi_{t-1} + A_{13}^2 \Delta x_{3t-1} + \varepsilon_{1t} . \end{aligned} \tag{6}$$

Defining $w_t = \begin{pmatrix} \Delta x_{1t} & \xi_t & x_{3t} \end{pmatrix}'$ the SVAR(2) can be expressed as

$$B_0 w_t = B_1 w_{t-1} + B_2 w_{t-2} + \varepsilon_t . \tag{7}$$

Partitioning (7) into the form conformable with the partition used in (5), the first m equations will be

$$\begin{aligned} &B_{11}^0 \Delta x_{1t} + B_{12}^0 \xi_t + B_{13}^0 x_{3t} \\ &= B_{11}^1 \Delta x_{1t-1} + B_{12}^1 \xi_{t-1} + B_{13}^1 x_{3t-1} + B_{11}^2 \Delta x_{1t-2} + B_{12}^2 \xi_{t-2} + B_{13}^2 x_{3t-2} + \varepsilon_{1t} \end{aligned}$$

which can be written as,

$$\begin{aligned} &B_{11}^0 \Delta x_{1t} + B_{12}^0 \Delta \xi_t + B_{13}^0 \Delta x_{3t} \\ &= B_{11}^1 \Delta x_{1t-1} + (B_{12}^1 + B_{12}^2 - B_{12}^0) \xi_{t-1} + (B_{13}^1 + B_{13}^2 - B_{13}^0) x_{3t-1} + B_{11}^2 \Delta x_{1t-2} - B_{12}^2 \Delta \xi_{t-1} - B_{13}^2 \Delta x_{3t-1} + \varepsilon_{1t} . \end{aligned} \tag{8}$$

Comparing (8) with (6), we get

[3] It may be necessary to take care in setting the system up to ensure that β_2' is non-singular. To take an example, suppose there are three I(1) variables in the system with cointegrating vector $\beta' = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$. Then, if we select the first and second equations as the two whose structural shocks have permanent effects, $\beta_2' = 0.$ So we would need to choose either the first and third or the second and third as the two variables whose associated equations have permanent shocks. In the former case $\beta_2' = -1$ and, in the latter, $\beta_2' = 1$ and so are non-singular.

$$\alpha_1^* = -(B_{12}^1 + B_{12}^2 - B_{12}^0), \qquad (9)$$

$$\delta_1^* = -(B_{13}^1 + B_{13}^2 - B_{13}^0). \qquad (10)$$

Now (7) can be written in lag operator form as

$$B(L)w_t = \varepsilon_t,$$

where $B(L) = B_0 - B_1 L - B_2 L^2$ and L is the lag operator. It then follows that the moving average representation will be

$$w_t = B(L)^{-1}\varepsilon_t \;\; = C(L)\varepsilon_t \qquad (11)$$

where $C(L) = C_0 + C_1 L + C_2 L^2 + C_3 L^3 + \ldots$. Hence $C(1) = B(1)^{-1}$ implies that

$$C(1)B(1) = I_n \,. \qquad (12)$$

By assumption shocks to the error correction terms ξ_t are transitory, so it must be the case that $C_{12}(1) = 0$, where $C(1)$ is partitioned analogously to the partitioned matrices in (8). *When* shocks to the I(0) variables are transitory, it is the case that $C_{13}(1) = 0$. These both place restrictions on the B matrices. To determine what they are multiply the first row of $C(1)$ with the second column of $B(1)$ to obtain the equation

$$C_{11}(1)B_{12}(1) + C_{12}(1)B_{22}(1) + C_{13}(1)B_{32}(1) = 0_{12}, \qquad (13)$$

where 0_{12} is an $m \times p$ null matrix. Under the restrictions, (13) becomes $C_{11}(1)B_{12}(1) = 0_{12}$, from which it follows that $B_{12}(1) = 0_{12}$, since $C_{11}(1)$ has full rank m. But $B_{12}(1) = (B_{12}^0 - B_{12}^1 - B_{12}^2)$ so that $B_{12}(1) = 0_{12}$ means $\alpha_1^* = 0_{12}$ from (9). This is the Pagan and Pesaran result.

Similarly, multiplying the first row of $C(1)$ with the third column of $B(1)$ gives

$$C_{11}(1)B_{13}(1) + C_{12}(1)B_{23}(1) + C_{13}(1)B_{33}(1) = 0_{13}, \qquad (14)$$

where 0_{13} is an $m \times q$ null matrix. Using the same reasoning, it follows that $B_{13}(1) = 0_{13}$, and noting $B_{13}(1) = (B_{13}^0 - B_{13}^1 - B_{13}^2)$, this means $\delta_1^* = 0_{13}$ by (10). Thus, when the structural shocks coming from the I(0) variables are transitory, the m structural equations with the permanent shocks do not contain the levels of the I(0) variables, only their differences. This case is an extension of the Pagan and Pesaran result for the SVAR involving Δx_{1t} and $\Delta\xi_t$ that the levels of the EC terms were replaced by their differences in the structural equations for Δx_{1t}.

2.3. Shocks associated with I(0) variables are mixed

It is useful to look at a simple example in order to appreciate the fact that one needs to approach mixed shocks with care. To this end consider the system

$$\Delta y_t = \delta z_t + \varepsilon_{1t} \tag{15}$$

$$z_t = \gamma z_{t-1} + \varepsilon_{2t}, \tag{16}$$

where y_t is I(1), z_t is I(0) and the shocks ε_{1t} and ε_{2t} are white noise. Hence, using the Beveridge-Nelson decomposition to find the permanent component of y_t we have

$$y_t^p = y_t + E_t \sum_{j=1}^{\infty} \Delta y_{t+j} = y_t + E_t \sum_{j=1}^{\infty} (\delta z_{t+j} + \varepsilon_{1t+j}) = y_t + \frac{\gamma\delta}{1-\gamma} z_t \ ,$$

so that

$$\Delta y_t^p = \Delta y_t + \frac{\delta\gamma}{1-\gamma}\Delta z_t = \delta z_t + \varepsilon_{1t} + \frac{\delta\gamma}{1-\gamma}\Delta z_t$$

This shows that, unlike the traditional case treated in co-integration theory (with shocks being permanent and transitory) where Δy_t^p is white noise, now the change in y_t^p is generally serially correlated, owing to the impact of the mixed shocks on the permanent component. Only if the second shock is transitory $(\delta = 0)$ will the change in y_t^p be white noise i.e. ε_{1t}.

Now, setting $\delta = 0$ would eliminate z_t from the first structural equation (15). It is however possible to allow z_t to appear in this equation, and also to make the second shock ε_{2t} transitory, by specifying the equation as $\Delta y_t = \delta\Delta z_t + \varepsilon_{1t}$. Then

$$y_t^p = y_t + E_t \sum_{j=1}^{\infty} (\delta\Delta z_{t+j} + \varepsilon_{1t+j}) = y_t + \delta(z_t^p - z_t) = y_t - \delta z_t,$$

using the facts that $z_t^p = z_t + E_t \sum_{j=1}^{\infty} \Delta z_{t+j}$, $z_t^p = 0$ (z_t is I(0) with zero mean) and ε_{1t} is white noise.

Therefore,

$$\frac{\partial y_t^p}{\partial \varepsilon_{2t}} = \frac{\partial y_t}{\partial \varepsilon_{2t}} - \delta \frac{\partial z_t}{\partial \varepsilon_{2t}} = \delta - \delta = 0,$$

that is ε_{2t} has only transitory effects. Of course, this is the result that we established more generally in the previous sub-section viz. that the I(0) variables have to be entered in differences if extra shocks in the system coming from the introduction of the I(0) variables are to have transitory effects.

It would generally be the case that the SVAR specified by empirical researchers would involve Δy_t and z_t, and so it is clear that such a SVAR would not incorporate a structural equation that had Δz_t on the RHS. One needs to modify existing SVAR programs to get that effect i.e. to make the extra shocks from the I(0) variables have transitory effects. Notice that if one had used an SVAR with Δy_t and Δz_t, this would change the specification of the second equation to an AR(1) in Δz_t. However this is a very different specification.

In an earlier section we noted that often it had been suggested that I(0) variables can be handled by using the idea of pseudo co-integrating vectors. Would this approach give a correct estimate of the permanent component of y_t in the simple system above? In this system there is no co-integration so that β will be the pseudo co-integrating vector $\beta' = (0 \quad 1)$. So the first issue is what the pseudo-SVECM is like. In this approach z_t is treated as if it is I(1), in which case one pseudo-SVECM might be

$$\Delta y_t = \delta \Delta z_t + \varepsilon_{1t} \tag{17}$$

$$\Delta z_t = (\gamma - 1) z_{t-1} + \varepsilon_{2t} \tag{18}$$

but, as we noted above, in this formulation the shocks ε_{2t} must be transitory and cannot be mixed.

An alternative pseudo-SVECM then is

$$\Delta y_t = \delta z_t + \varepsilon_{1t}$$

$$\Delta z_t = (\gamma - 1) z_{t-1} + \varepsilon_{2t}$$

which has the pseudo-VECM form

$$\Delta y_t = \delta \gamma z_{t-1} + e_{1t} \tag{19}$$

$$\Delta z_t = (\gamma - 1) z_{t-1} + e_{2t} \tag{20}$$

leading to $\alpha = \begin{pmatrix} \delta\gamma \\ \gamma - 1 \end{pmatrix}$. In the standard approach to extracting permanent components, and there is a single permanent shock, the latter becomes a multiple of $\alpha'_{\perp} e_t$, where e_t are the pseudo-VECM residuals and $\alpha'\alpha_{\perp} = 0$. For the pseudo VECM in (19) and (20), $e_{1t} = \delta\varepsilon_{2t} + \varepsilon_{1t}$ and $\varepsilon_{2t} = e_{2t}$. Consequently, using $\alpha'_{\perp} = \begin{pmatrix} 1 & -\delta\gamma/(\gamma-1) \end{pmatrix}$ and we see that the implied permanent shock would be proportional to $\delta\varepsilon_{2t} + \varepsilon_{1t} - [\delta\gamma/(\gamma-1)]\varepsilon_{2t}$, and so it would depend on ε_{2t}. However, because the standard formulation from co-integration also implies that Δy_t^p is proportional to $\alpha'_{\perp} e_t$, using the pseudo EC approach would predict that Δy_t^p is white noise, whereas we have already seen in this case that it is serially correlated. Hence the approach gives an incorrect estimate of the permanent component.

2.4. General formula for computation of permanent components of I(1) series when there is a mixture of I(1) and I(0) series

We will consider the following VAR system

$$\Delta y_t = A_1 \Delta y_{t-1} + G z_{t-1} + e_{1t} \tag{21}$$

$$z_t = F z_{t-1} + \Phi \Delta y_{t-1} + e_{2t} \tag{22}$$

where z_t has both the EC terms and the I(0) variables in it. To rationalize (21) and (22) think of the case where all variables y_t are I(1) and there is co-integration. Then we would have $\Delta y_t = A_1 \Delta y_{t-1} + \alpha\beta' y_{t-1}$ and this can be written as $\beta' \Delta y_t = \Delta ec_t = \beta' A_1 \Delta y_{t-1} + \beta' \alpha ec_{t-1}$, thereby giving an equation that has the form $ec_t = (I + \beta'\alpha) ec_{t-1} + \beta' A_1 \Delta y_{t-1}$. So this has the structure of (22) with ec_t being included in z_t. Although we are working with a first order system, higher order systems can be handled simply by reducing them to a first order form in the standard way.

Now the permanent component of y_t is $y_t^p = y_t + E_t \sum_{j=1}^{\infty} \Delta y_{t+j}$ so we need to look at the second term. This will be

$$E_t \sum_{j=1}^{\infty} \Delta y_{t+j} = E_t \sum_{j=1}^{\infty} (A_1 \Delta y_{t-1+j} + G z_{t-1+j} + e_{1t+j}). \tag{23}$$

Now let us consider $L_t = \sum_{j=1}^{M} \Delta y_{t+j-1}$ and define $K_t = \sum_{j=1}^{M} \Delta y_{t+j}$. Then it is clear that $L_t = K_t + \Delta y_t - \Delta y_M$. Thus, as $M \to \infty,\ E_t(L_t) = E_t(K_t) + \Delta y_t$. Consequently, when $M \to \infty,$ we can write (23) above as

$$E_t K_t = (A_1 E_t K_t + A_1 \Delta y_t) + G E_t \sum_{j=1}^{\infty} z_{t-1+j} \tag{24}$$

$$\therefore\ E_t K_t = (I - A_1)^{-1} A_1 \Delta y_t + (I - A_1)^{-1} G E_t \sum_{j=1}^{\infty} z_{t-1+j} \tag{25}$$

This makes sense since, if $G = 0,$ then the shocks e_{2t} have no permanent effects.

Now from (22)

$$E_t \sum_{j=1}^{\infty} z_{t-1+j} = E_t (F \sum_{j=1}^{\infty} z_{t-2+j} + \Phi \sum_{j=1}^{\infty} \Delta y_{t-2+j} + \sum_{j=1}^{\infty} e_{2t+j-1}) = F E_t \sum_{j=1}^{\infty} z_{t-2+j} + \Phi E_t \sum_{j=1}^{\infty} \Delta y_{t-2+j} + e_{2t}. \tag{26}$$

Using the same methodology as above we let $Q_t = \sum_{j=1}^{M} z_{t-1+j}$, $P_t = \sum_{j=1}^{M} z_{t-2+j}$, so that $P_t = Q_t + z_{t-1} - z_{t+M}$, enabling us to express (26) as

$$E_t Q_t \quad = F E_t Q_t + F z_{t-1} + E_t \Phi \sum_{j=1}^{\infty} \Delta y_{t-2+j} + e_{2t}$$

$$= F E_t Q_t + F z_{t-1} + \Phi E_t L_t + \Phi \Delta y_{t-1} + e_{2t}$$

$$\therefore\ E_t Q_t = (I - F)^{-1} (F z_{t-1} + \Phi E_t L_t + \Phi \Delta y_{t-1} + e_{2t})$$

$$= (I - F)^{-1} (F z_{t-1} + \Phi E_t K_t + \Phi \Delta y_t + \Phi \Delta y_{t-1} + e_{2t})$$

Now replacing $\sum_{j=1}^{\infty} z_{t-1+j}$ by Q_t in (25) when $M \to \infty$, we get

$$E_t K_t = (I - A_1)^{-1} A_1 \Delta y_t + (I - A_1)^{-1} G E_t Q_t,$$

whereupon using the expression for $E_t Q_t$ yields

$$E_t K_t = (I - A_1)^{-1} A_1 \Delta y_t + (I - A_1)^{-1} G (I - F)^{-1} (F z_{t-1} + \Phi E_t K_t + \Phi \Delta y_t + \Phi \Delta y_{t-1} + e_{2t}).$$

Defining $R = (I - A_1)^{-1} G (I - F)^{-1}$ we have

$$E_t K_t = (I - A_1)^{-1} A_1 \Delta y_t + R(F z_{t-1} + \Phi E_t K_t + \Phi \Delta y_t + \Phi \Delta y_{t-1} + e_{2t})$$

and so

$$E_t K_t = (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi]\Delta y_t + (I - R\Phi)^{-1} R z_t$$

This gives us $y_t^p = y_t + E_t K_t$.

Let us look at the simple examples in the beginning of section 2.3. The first model has the form

$$\Delta y_t = \delta z_t + \varepsilon_{1t} = \delta\gamma z_{t-1} + \varepsilon_{1t} + \delta\varepsilon_{2t} = \delta\gamma z_{t-1} + e_{1t}$$

$$z_t = \gamma z_{t-1} + \varepsilon_{2t} = \gamma z_{t-1} + e_{2t}$$

Then $G = \delta\gamma,\ F = \gamma,\ A_1 = 0,\ \Phi = 0,\ R = G(I - F)^{-1}$ so that

$$E_t K_t = R z_t = \frac{\delta\gamma}{1-\gamma} z_t$$

as we found earlier. Thus the second shock has a permanent effect.

Looking at the second model where

$$\Delta y_t = \delta\Delta z_t + \varepsilon_{1t} = \delta(\gamma - 1) z_{t-1} + \varepsilon_{1t} + \delta\varepsilon_{2t} = \delta(\gamma - 1) z_{t-1} + e_{1t}$$

$$z_t = \gamma z_{t-1} + \varepsilon_{2t} = \gamma z_{t-1} + e_{2t}$$

then $G = \delta(\gamma - 1),\ F = \gamma,\ A_1 = 0,\ \Phi = 0,\ R = G(I - F)^{-1}$ and

$$E_t K_t \quad = \ R z_t = -\delta z_t$$

$$\therefore\ \frac{\partial y_t^p}{\partial e_{2t}} = \frac{\partial y_t}{\partial e_{2t}} + \frac{\partial E_t K_t}{\partial e_{2t}} = \delta - \delta = 0,$$

which shows the second shock only has a transitory effect, as found previously.

3. An illustration of the treatment of mixed shocks in SVARs

In an influential paper, Peersman (2005) estimated an SVAR model of four variables to investigate the role played by the underlying structural shocks in the early millennium slowdown experienced in the United States and Europe. The VAR consisted of the oil price (o_t), output (y_t), consumer prices

(p_t) (all in log levels) and the short-term nominal interest rate (s_t). The oil price, output and consumer prices are treated as I(1) variables and the short-term interest rate as an I(0) variable. There was no evidence for a cointegrating relation among the I(1) variables. In view of these properties of the data, Peersman followed common practice and specified an SVAR in the first difference of the I(1) variables and in the level of the stationary variable.

To exactly identify the SVAR, Peersman imposed two long-run and four contemporaneous restrictions. Under these restrictions, the structural shock to oil prices was interpreted as an oil price shock, to output as a supply shock, to consumer prices as a demand shock and to the interest rate as a monetary policy shock. The two long-run restrictions are that the demand and monetary policy shocks have a zero long-run effect on output, and these distinguish those shocks from the oil price and supply shocks. In order to distinguish the monetary policy shock from the demand shock, Peersman imposed the restriction that the monetary policy shock has a zero contemporaneous effect on output. Finally, he assumed that the change in oil prices does not depend on the contemporaneous change in output, consumer prices and the interest rate. These serve to differentiate the supply shock from the oil price shock and also imply that supply, demand and monetary policy shocks have a zero contemporaneous effect on oil prices. Here the monetary policy shock is, in our terminology, a mixed shock, as it arises from the introduction of the I(0) interest rate variable and it is permitted to have a long-run effect on some of the I(1) variables, specifically, consumer and oil prices.

The SVAR was specified with three lags and each equation included a constant and a time trend. Peersman estimated the SVAR by maximum likelihood methods for the sample 1980Q1 – 2002Q2. Figure 1(a) of his paper (2005, p.189) shows the impulse responses of the variables to the structural shocks out to a 28 quarter horizon (the differences between 28 and 200 quarter horizons are small, so we will use the 28 quarter results as showing the long-run responses). An inspection of this figure reveals several features. First, in response to a monetary policy shock which raises the short-term interest rate, both consumer prices and oil prices fall over all horizons i.e. there are no ”price puzzles”. While output increases by only a small amount initially, it then falls over the next four quarters, after which it starts to gradually recover to its level prior to the monetary policy shock. Third, the monetary policy shock has a long-run effect on relative prices since oil prices fall proportionately more than consumer prices (2% compared with 0.3%) at the 28 quarter horizon. Fourth, the demand shock has a long-run effect on relative prices. Oil prices increase by 3% in response to a positive demand shock at the 28 quarter horizon while consumer prices increase by only 0.3%. While there are no “output” and “price” puzzles in the results, the monetary policy and demand shocks have a long-run effect on relative prices.[4] Because it is standard in most economic models for demand and monetary policy shocks to have only transitory effects on relative prices and output so that in the long-run relative prices and output are unaffected by these shocks, we would expect that the SVAR should also be designed to have such properties. We now turn to how this is to be done.

3.1. Design of the SVAR

To arrive at a SVAR with the long-run properties just mentioned, we begin by replacing the price of oil with the *relative* price of oil, defined as $\varsigma_t = o_t - p_t$. This is also an I(1) variable, as Peersman found no co-integration between the I(1) variables. The resulting SVAR is:[5]

$$\Delta\varsigma_t = a_{11}^1\Delta\varsigma_{t-1} + a_{12}^0\Delta y_t + a_{12}^1\Delta y_{t-1} + a_{13}^0\Delta p_t + a_{13}^1\Delta p_{t-1} + a_{14}^0 s_t + a_{14}^1 s_{t-1} + \varepsilon_{1t} \quad (27)$$

$$\Delta y_t = a_{21}^0\Delta\varsigma_t + a_{21}^1\Delta\varsigma_{t-1} + a_{22}^1\Delta y_{t-1} + a_{23}^0\Delta p_t + a_{23}^1\Delta p_{t-1} + a_{24}^0 s_t + a_{24}^1 s_{t-1} + \varepsilon_{2t} \quad (28)$$

$$\Delta p_t = a_{31}^0\Delta\varsigma_t + a_{31}^1\Delta\varsigma_{t-1} + a_{32}^0\Delta y_t + a_{32}^1\Delta y_{t-1} + a_{33}^1\Delta p_{t-1} + a_{34}^0 s_t + a_{34}^1 s_{t-1} + \varepsilon_{3t} \quad (29)$$

$$s_t = a_{41}^0\Delta\varsigma_t + a_{41}^1\Delta\varsigma_{t-1} + a_{42}^0\Delta y_t + a_{42}^1\Delta y_{t-1} + a_{43}^0\Delta p_t + a_{43}^1\Delta p_{t-1} + a_{44}^1 s_{t-1} + \varepsilon_{4t} \quad (30)$$

The four long-run restrictions we impose are that demand and monetary policy shocks have a zero long-run effect on relative prices and output. With respect to relative prices, the restrictions are, respectively,

$$a_{13}^0 + a_{13}^1 = 0, \quad a_{14}^0 + a_{14}^1 = 0, \quad (31)$$

and, with respect to output, they would be

$$a_{23}^0 + a_{23}^1 = 0, \quad a_{24}^0 + a_{24}^1 = 0. \quad (32)$$

[4] We estimated Peersman's SVAR by IV and replicated his results. We will later find some price and output puzzles in various SVARs we estimate. There are of course suggestions that these puzzles may not be so e.g. it has been argued that a rise in interest rates could increase the price level owing to increased working capital costs. However, mostly such results are regarded as abnormal, and so classified as puzzles. We will just follow the conventional approach here and classify rises in output and prices in response to monetary policy shocks as "puzzles".

[5] For ease of exposition, our development assumes an SVAR of order one which does not include deterministic terms. It can be easily generalised to the SVAR we actually estimate which, following Peersman, has three lags and a constant and time trend in each equation.

These enable demand and monetary policy shocks to be differentiated from relative oil price and supply shocks.[6]

We require two contemporaneous restrictions, one to separate demand from monetary policy shocks and the other to separate relative oil price from supply shocks. They are, respectively, that the demand and supply shock have a zero contemporaneous effect on the relative price of oil. These are the equivalent of two of Peersman's short-run restrictions, though now with respect to the relative oil price.

These restrictions can be imposed parametrically on (27)-(30). Let the (4×4) matrix of contemporaneous interactions among the variables be denoted by A_0, where the elements along the principal diagonal are unity, so the first structural equation is for the change in the relative oil price, the second for the change in output, the third for the change in consumer prices and the fourth for the interest rate. The relationship between the structural shocks and the reduced form errors (e_t) is given by $\varepsilon_t = A_0 e_t$. Let the element in the *i*th row and the *j*th column of A_0^{-1} be denoted as a_0^{ij}. Then the restriction that the demand shock has a zero contemporaneous effect on relative oil prices is expressed as $a_0^{13} = 0$. Because $e_t = A_0^{-1}\varepsilon_t$ the reduced form errors are linear combinations of the structural shocks, so that the restriction $a_0^{13} = 0$ means that the demand shock does not appear in the reduced form (VAR) error for relative oil prices. Consequently the residuals from the VAR equation for relative oil prices (e_{1t}) can be used as an instrument in the estimation of the consumer price equation. Similarly, the restriction that the supply shock has a zero contemporaneous effect on relative oil prices is $a_0^{12} = 0$, showing that the VAR relative oil price residuals can also be used as an instrument in the estimation of the output equation. The two contemporaneous restrictions together with the four long-run restrictions shown in (31) and (32) produce the correct number of restrictions to identify the SVAR parameters.

3.2. Estimation

Imposing the two long-run restrictions in (32) on (28), the equation for the change in output becomes

$$\Delta y_t = a_{21}^0 \Delta\varsigma_t + a_{21}^1 \Delta\varsigma_{t-1} + a_{22}^1 \Delta y_{t-1} + a_{23}^0 \Delta^2 p_t + a_{24}^0 \Delta s_t + \varepsilon_{2t}. \tag{33}$$

[6] To implement these restrictions, we note, for example, that $a_{24}^0 s_t + a_{24}^1 s_{t-1}$ can be expressed as $a_{24}^0 \Delta s_t + (a_{24}^0 + a_{24}^1) s_{t-1}$.

We estimate this equation using, as instruments, $\hat{e}_{1t}$, Δp_{t-1}, s_{t-1}, as well as $\Delta\varsigma_{t-1}$ and Δy_{t-1}. The next equation to estimate is the equation for the relative price of oil that is obtained by imposing the restrictions in (31) on (27). The resulting equation is

$$\Delta\varsigma_t = a_{11}^1 \Delta\varsigma_{t-1} + a_{12}^0 \Delta y_t + a_{12}^1 \Delta y_{t-1} + a_{13}^0 \Delta^2 p_t + a_{14}^0 \Delta s_t + \varepsilon_{1t}. \tag{34}$$

It is estimated using, as instruments, the residuals $\hat{\varepsilon}_{2t}$ from (33), along with Δp_{t-1}, s_{t-1}, $\Delta\varsigma_{t-1}$ and Δy_{t-1}. The next equation estimated is (29), the equation for consumer prices. Here the instruments are $\hat{\varepsilon}_{1t}$, $\hat{\varepsilon}_{2t}$, $\hat{e}_{1t}$, as well as $\Delta\varsigma_{t-1}$, Δy_{t-1}, Δp_{t-1} and s_{t-1}. Finally, the last equation estimated is (30), the interest rate equation. For this, we use the estimated residuals $\hat{\varepsilon}_{1t}$, $\hat{\varepsilon}_{2t}$ and $\hat{\varepsilon}_{3t}$ as well as $\Delta\varsigma_{t-1}$, $\Delta y_{t-1}, \Delta p_{t-1}$ and s_{t-1}, as the instruments.

3.3 Results

Figure 1 shows the impulse responses of the U.S. variables to the structural shocks out to a horizon of 28 quarters.[7] Note that the response of the oil price itself to a shock is simply the sum of the relative oil price and consumer price response to that shock. The identifying restrictions are apparent in the responses: the demand and monetary policy shocks have a zero long-run effect on relative oil prices and output, and the supply and demand shocks have a zero contemporaneous effect on relative oil prices.[8]

With this specification, however, there are “price” and “output” puzzles. In response to a monetary policy shock which raises the interest rate, consumer prices steadily rise and by 28 quarters have increased by 0.3%. The oil price initially falls by about 3%, so there is no “relative oil price” puzzle, and by 28 quarters it has increased by the same proportionate amount as consumer prices, leaving long-run relative oil prices unchanged. Output initially rises by around 0.3% following the monetary policy shock so there is an “output puzzle”.

We estimated several SVARs under other combinations of two zero contemporaneous restrictions while maintaining the four long-run restrictions. In all the SVAR’s, at least one puzzle was

[7] The responses at 28 quarters are sufficient to show the long-run as they are indistinguishable from those at much longer horizons (we generated responses out to 200 quarters and saw no discernible differences). The impulse responses are shown together with their one standard error bands based on 1000 bootstrapped draws. In the bootstrap, the forecast values and re-sampled residuals from the reduced-form VAR model estimated with actual data were used to construct artificial time series for each variable.

[8] As a check on our results, we also estimated the model using the *short and long procedure* in RATS Version 8.2. The RATS numerical procedure confirmed the results from IV estimation and the numerical differences between the two sets of impulse responses were slight.

apparent in the responses. When there was a consumer price puzzle, there was no oil price puzzle and vice-versa, and it was only in specifications which restricted the contemporaneous response of output to the monetary policy shock to zero that the output puzzle disappeared.[9] It appears that, once demand and monetary policy shocks are restricted to have only transitory effects on relative prices, "puzzles" emerge. Once these shocks are allowed to have permanent effects on relative prices, the "puzzles" disappear. Our experience in other applications is that this is a common phenomenon and it should force empirical researchers to justify why they allow nominal shocks to have long-run effects on real variables and relative prices.

4. Sign Restrictions with Mixed Shocks

In addition to the parametric approach, Peersman chose to use the sign restrictions methodology, developed by Faust (1998), Uhlig (2005) and Canova and De Nicoló (2002), to identify the structural shocks. The method starts by obtaining an initial set of shocks that are uncorrelated. Peersman followed traditional practice and obtained these from a recursive model. While this restricts the contemporaneous impacts of the initial shocks, it leaves the long-run impacts unrestricted. In our application, we specify the initial model to preserve the two long-run restrictions that the third and fourth shocks have a zero long-run effect on output and then make the model recursive. In this way, the initial shocks are orthogonal and have the property that the third and fourth shocks do not have a long-run impact on output.

To describe our initial model, we will refer to (27)-(30) for the variable numbers. However, now the relative price of oil has to be replaced by the price of oil, as we are re-considering the results from Peersman's original model with shocks now being identified using sign restrictions.[10] The first equation we set up to generate shocks that are to be the basis of the sign restrictions approach is for output i.e. (33). This has imposed on it the two long-run zero restrictions. But we need a further restriction, and that involves assuming oil prices are ordered after output, so that $a_{21}^{0}=0.$ The next equation is for the change in consumer prices (29), and here we assume that oil prices and interest rates are ordered after the general price level, thereby generating the restrictions $a_{31}^{0}=0$ and $a_{34}^{0}=0.$ The oil price equation (27) uses the restriction $a_{14}^{0}=0,$ that is interest rates are ordered after the oil price, Finally no restrictions are placed on the interest rate equation (30). The model just described is then estimated by IV. In estimation of the consumer price equation, $\hat{\varepsilon}_{2t}$ is used as an

[9] This pattern emerged in all specifications including ones that left unrestricted the contemporaneous effect of all the shocks on the relative price of oil.

[10] Again, in the actual application, we follow Peersman and estimate a SVAR with three lags and a constant and time trend in each equation.

instrument; in estimation of the oil price equation, $\hat{\varepsilon}_{2t}$ and $\hat{\varepsilon}_{3t}$ are used as instruments; and in the interest rate equation, $\hat{\varepsilon}_{2t}$, $\hat{\varepsilon}_{3t}$ and $\hat{\varepsilon}_{1t}$ are used as instruments.

In sign restrictions, the initial shocks from the model just described are normalized to have unit variance so they become $\hat{\varepsilon}^{*}_{i,t} = (\hat{\varepsilon}_{i,t} / \hat{\sigma}_i)$, $i = 1,2,3,4$ and are $i.i.d(0, I_4)$. We focus on the group $\hat{\varepsilon}^{*}_{R,t} = (\hat{\varepsilon}^{*}_{3t} \quad \hat{\varepsilon}^{*}_{4t})'$ as these are restricted to have a zero long-run effect on output.[11] The next step is to linearly combine these shocks to form a new set of shocks $\hat{\eta}^{*}_{R,t} = Q\hat{\varepsilon}^{*}_{R,t}$, where the (2×2) matrix Q is the Givens matrix

$$\begin{bmatrix} \cos\theta_k & -\sin\theta_k \\ \sin\theta_k & \cos\theta_k \end{bmatrix}, \quad \theta_k \in (0, \pi),$$

with the property that $Q'Q = QQ' = I_2$. The Q matrix depends on a 'draw' of θ_k and, in sign restrictions, the number of draws is large.[12] Note that the new shocks are uncorrelated with each other.

Now let the (4×2) matrix $C_{R,j}$ denote the responses at horizon *j* of the variables to a one unit innovation in each of the shocks in $\hat{\varepsilon}^{*}_{R,t}$. Then, for a given draw of the Givens matrix, the responses to a one unit innovation in each of the new shocks, $\hat{\eta}^{*}_{R,t}$, is $C_{R,j}Q'$. Note that the long-run response of output to $\hat{\eta}^{*}_{R,t}$ is zero since both elements of the second row of $C_{R,\infty}$ are zero. Sign restrictions are now used to distinguish between the two shocks in $\hat{\eta}^{*}_{R,t}$.[13] The restrictions we use are taken from Peersman. A positive monetary policy shock raises the interest rate and has a non-positive effect on oil prices, output, and consumer prices. In contrast, if all the responses are non-negative, it is treated as a positive demand shock.[14]

We found that 0.578% of the draws satisfied the sign restrictions for demand and monetary policy shocks. This success rate is a little lower than what Peersman reported (1 in 130 or 0.769%).

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[11] Separating the shocks into appropriate groups and applying sign restrictions to each group is the approach taken by Fry and Pagan (2011) for co-integrated systems in which there are both permanent and transitory shocks. As we are making finer distinctions among the shocks, it is natural to adopt a similar approach here, so that the new shocks will retain the features of the initial shocks.

[12] In our application, $\theta_k = k(\pi / 500{,}000),\ k = 0,1,2,\ldots,500{,}000$.

[13] They could also be used to separate the shocks in the group $\hat{\eta}^{*}_{U,t} = (\hat{\eta}^{*}_{1t} \quad \hat{\eta}^{*}_{2t})'$ but that is not our focus.

[14] In line with Peersman, the time period over which the sign restrictions are binding is for four quarters on the responses of output and consumer prices and only on the instantaneous response of oil prices and the interest rate.

In both cases however these low retention rates suggest that the data does not support the sign restrictions. Based on the successful draws, figure 2 reports the median (50th fractile) responses to unit shocks. Demand and monetary policy shocks have a zero long-run effect on output by design but they clearly have long-run effects on the relative price of oil.

In our signs approach, care needs to be exercised in formulating the initial recursive model. Suppose we had decided to order the oil price before output. Then this would mean that the initial third and fourth shocks have a zero contemporaneous effect on oil prices. Now these two shocks have the requisite zero long-run effects so we linearly combine them together to form new shocks. But this must mean that any new shocks have a zero contemporaneous effect on oil prices. It does not seem reasonable to constrain the demand and monetary shocks to always have such effects. Consequently, this led us to adopt the ordering described where oil prices came after output and the general price level.

5. Conclusion

The inclusion of I(0) variables in structural econometric models introduces additional shocks which do not fit neatly into the traditional classification of shocks in I(1) systems as permanent or transitory. We augment this classification by describing shocks associated with the I(0) variables as either mixed (having a non-zero long-run effect on at least one I(1) variable) or transitory (having a zero long-run effect on all I(1) variables).

We showed that several well-known results follow through to settings which include I(0) variables provided the shocks associated with them are transitory. The Pagan and Pesaran (2008) result about the nature of structural equations with permanent shocks can be extended to this case, and familiar results associated with the Beveridge-Nelson decomposition and from cointegration analysis apply. However, if the shocks are mixed, familiar results may no longer apply. We show that it is no longer the case that the change in the permanent component of an I(1) series defined by the Beveridge-Nelson decomposition would be white noise. We also show that a standard approach to handling both I(1) and I(0) variables coming from the co-integration would no longer give the correct answer for the permanent component when shocks are mixed. Consequently, we derive a general formula for computing the permanent component when shocks are mixed.

We then turned to some applications, using as the vehicle Peersman's influential SVAR which features a mixed shock. The latter arises from the presence of an I(0) interest rate variable and is mixed because it is allowed to have a long-run effect on oil and consumer prices, both of which are I(1). In Peersman's SVAR, there are no price or output puzzles but there is monetary non-neutrality, since the mixed shock affects relative prices in the long-run. When the mixed shock is made transitory output and price puzzles emerge. We conclude that the absence of price and output puzzles in Peersman's VAR comes about because he allows the mixed shock to have a long-run effect on relative prices; that is, the absence of puzzles comes at the cost of monetary neutrality. Finally, we show how to apply sign restrictions to the SVAR for which the two long-run zero restrictions of Peersman are maintained.

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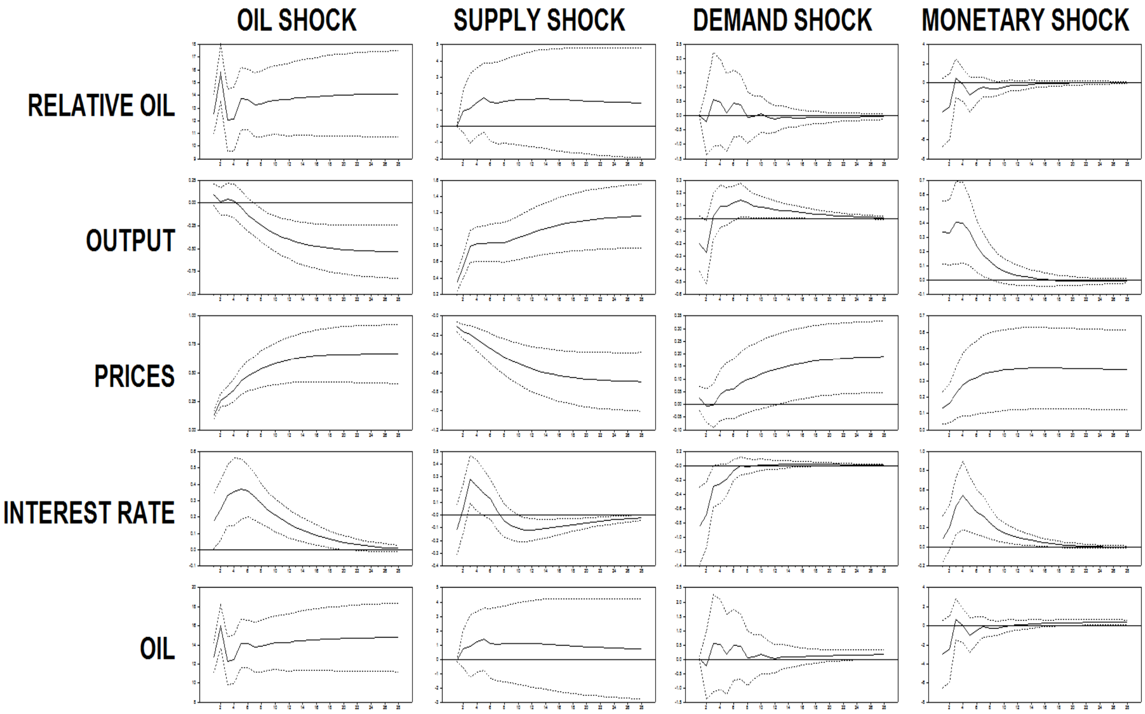

Figure 1. Impulses responses from relative price model

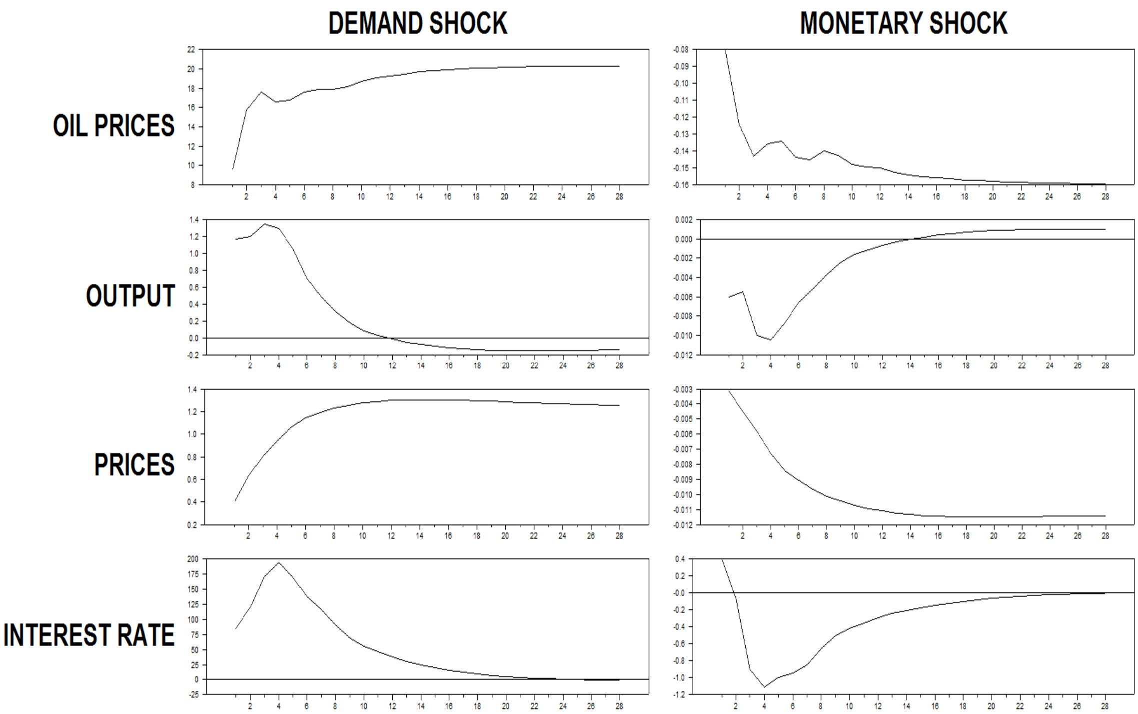

Figure 2. Impulse responses based on signs