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Abstract

This note shows that the common practice of adding on measurment errors or "errors in variables" when estimating DSGE models can imply that there is a lack of co-integration between model and data variables and also between data variables themselves. An analysis is provided of what the nature of the measurement error would be if it was desired to ensure co-integration. It is very unlikely that it would be the white noise shocks that are commonly used.

1 Introduction

Many applications of DSGE models use data measured as growth rates of some variables such as GDP, nominal exchange rates and price levels.[1] Examples of policy models would be the EDO model of the Federal Reserve - Chung et. al. (2010) - and the Multi-sector model of the Reserve Bank of Australia - Rees et. al. (2016). When estimation of the parameters is performed it is assumed that there is a discrepancy between model growth

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[1] This is also true if the data is measured with filters to produce an output gap. In those cases the filtered data will be weighted averages of growth rates in GDP. The difficulties we describe in this note also apply when filtered data is used, although the analysis is more complex.

variables and the data on them, and these discrepancies are often described as "errors in variables shocks" or "measurement error shocks".[2] The purpose of this note is to explore what the impact of these "errors in variables shocks" is. Section 2 shows that they generally imply that there is a lack of co-integration between the *levels* of the model variables and the corresponding data in levels, but also that there is a lack of co-integration between the data variables themselves. After showing this in a simple way sections 3 and 4 turn to the question of what happens when the model implies some co-integration between the model level variables, while the data may imply exactly the same or more co-integration than is implied by the model. In both cases one can make the model and data variables co-integrate by using time differences in "errors in variables shocks" as the augmenting mechanism. However this is at the expense of using an incorrect description of what the correct " errors in variables shocks" should be. It is rarely the case that one can treat the "errors in variables shocks" as white noise, as is typically done in most applied studies. The exception to that occurs if there is no co-integration in the data. So using white noise shocks is making the presumption that the data lacks co-integration. Whether this is the modeller's intention is the question mark of the title of the paper.

2 Case 1: The Most Common Approach

We start with a simple situation where there are three variables in both the model and the data. These are Δz^*_{Dt} = data on foreign GDP growth; Δz_{Dt} =data on domestic GDP growth and Δc_{Dt} =data on domestic consumption growth, where $D = data$. As well we have DSGE model variables (M = model) Δz^*_{Mt} , Δz_{Mt} and Δc_{Mt}. In the DSGE model there is a log level of technology process a_t which follows a pure random walk $a_t = a_{t-1} + \xi_t$, where ξ_t are white noise innovations that have zero mean and variance σ^2. This produces unit roots in the logs of the GDP and consumption processes and these co-integrate with a_t. Then the assumptions often made when estimating the DSGE model with the growth rate data have the form

[2] I have never been happy with this description. Basically what these shocks do is to measure the extent to which the model fails to track the data, and therefore Fukac and Pagan (2011) called them "tracking shocks".

$$\begin{aligned}
\Delta z^*_{Dt} &= \Delta z^*_{Mt} + \varepsilon_{1t} \\
\Delta z_{Dt} &= \Delta z_{Mt} + \varepsilon_{2t} \\
\Delta c_{Dt} &= \Delta c_{Mt} + \varepsilon_{3t},
\end{aligned}$$

where the ε_{jt} are white noise innovations that are uncorrelated with each other. To see the relation between the data and model *level* variables assume initial conditions are zero so that

$$\begin{aligned}
z^*_{Dt} - z^*_{Mt} &= \sum_{k=1}^{t} \varepsilon_{1k} \\
z_{Dt} - z_{Mt} &= \sum_{k=1}^{t} \varepsilon_{2k} \\
c_{Dt} - c_{Mt} &= \sum_{k=1}^{t} \varepsilon_{3k}.
\end{aligned}$$

Hence

$$z^*_{Dt} - z_{Dt} - (z^*_{Mt} - z_{Mt}) = \sum_{k=1}^{t} \varepsilon_{1k} - \sum_{k=1}^{t} \varepsilon_{2k} \tag{1}$$

Now within the model there is co-integration between z_{Mt} and z^*_{Mt} since they both co-integrate with the log level of technology a_t so that, using (1),

$$\begin{aligned}
z^*_{Dt} - z_{Dt} &= (z^*_{Mt} - z_{Mt}) + \sum_{k=1}^{t} \varepsilon_{1k} - \sum_{k=1}^{t} \varepsilon_{2k} \\
&= I(0) + \sum_{k=1}^{t} (\varepsilon_{1k} - \varepsilon_{2k})
\end{aligned}$$

But this must mean that $z^*_{Dt} - z_{Dt}$ is $I(1)$ unless $\sum_{k=1}^{t}(\varepsilon_{1k} - \varepsilon_{2k})$ is $I(0)$, which cannot happen unless $\varepsilon_{1t} = \varepsilon_{2t}$.We can see the same thing if we ask whether c_{Dt} and z_{Dt} co-integrate. c_t co-integrates with a_t and so c_{Mt} and z_{Mt} co-integrate but this is not true of $c_{D,t}$ and $c_{M,t}$.

So whenever the data is measured as growth rates in a variable z_t, and an "error in variable shock" is added into the observation equation, it implies there is no co-integration between the data and model variables. Moreover, if more than one I(1) variable is being treated in the same way, then this implies no co-integration between the variables in the data. This is a strong assumption and one that can be tested. Of course if there is no co-integration in the data then the standard method of adding on white noise "errors in variables shocks" will be appropriate. The question then arises of what to do when there is co-integration in the data.

3 Case 2: Both Data and Model Have Co-integration of the Same Form

We now look at the case where the model has some co-integrating vectors and the data co-integrates with the same ones. It is useful to be more specific initially so as to understand the issues. Therefore take the basic RBC model as the DSGE one, where there is a unit root in the log of technology a_t. Then this is log-linearized and model variables are expressed as deviations from a_t. Consequently the variables solved for in the model will be $\tilde{c}_t = c_t - a_t, \tilde{\imath}_t = i_t - a_t, \tilde{k}_t = k_t - a_t$ and $\tilde{y}_t = y_t - a_t$, where c_t,i_t, k_t and y_t are the logs of consumption, investment, the capital stock and output. Then $\tilde{c}_t, \tilde{\imath}_t, \tilde{k}_t$ and $\tilde{y}_t$ will be $I(0)$ and so c_t, i_t, k_t and y_t co-integrate with a_t. Alternatively, we can partially express this as y_t co-integrating with c_t, i_t and k_t because the co-integrating relations with a_t imply that $(c_t - y_t), (i_t - y_t)$ and $(k_t - y_t)$ are $I(0)$. Finally, there is a further co-integrating relation due to $(y_t - a_t)$ being $I(0)$.

Now a_t is not an observed variable and so the DSGE model will therefore have *one more co-integrating relation than the VECM in observed data* would have. Thus, if there are n observed $I(1)$ variables z_t^D, the corresponding model variables will be z_t^M, and the DSGE model can be expressed as a VECM of the form[3]

$$\Delta z_t^M = \delta\gamma' z_{t-1}^M + \psi(z_{nt}^M - a_t) + e_t^M, \tag{2}$$

where γ are the common co-integrating vectors, and we have chosen to normalize the extra co-integrating relation using the $n'th$ variable z_{nt}.[4] Now suppose that the data is generated by

[3] In Christensen et. al. (2011) an algorithm is given for converting DSGE model output into a VECM representation.

[4] The model and data VAR errors will be assumed to be white noise processes i.e. they are innovations but only need to be $I(0)$.

$$\Delta z_t^D = \delta\gamma' z_{t-1}^D + e_t^D. \tag{3}$$

Then, subtracting (2) from (3), gives

$$\Delta z_t^D - \Delta z_t^M = \delta\gamma'(z_{t-1}^D - z_{t-1}^M) - \psi(z_{nt}^M - a_t) + e_t^D - e_t^M.$$

Defining the term $\xi_t = z_t^D - z_t^M$ this evolves as

$$\Delta\xi_t = \delta\gamma'\xi_{t-1} - \psi(z_{nt}^M - a_t) + e_t^D - e_t^M,$$

and this will imply that ξ_t is $I(0)$. Consequently, there will be co-integration between the model and the data. Moreover, if we define $\Delta z_t^D - \Delta z_t^M = \eta_t$, it is clear that the "errors in variables shocks" η_t that reconcile data and model growth rates would need to be

$$\eta_t = \delta\gamma'\xi_{t-1} - \psi(z_{nt}^M - a_t) + e_t^D - e_t^M, \tag{4}$$

and it will be impossible for the vector η_t to be white noise, as is generally assumed. In fact due to the composite nature of the error term it will be a VARMA process. Notice the presence of the error correction terms in (4) and it is this that results in the co-integration.

Now one might set up the shock process as in (4), but suppose we simply want to ensure co-integration between data and model level variables. If we make the shock reconciling growth rates η_t a white noise process, v_t, then this would mean that $\Delta(z_t^D - z_t^M) = \eta_t = v_t$, and so there would be no co-integration between model and data level variables . However, by setting $\eta_t = \Delta v_t$ we would ensure co-integration, even though the true η_t that is needed to reconcile the growth rates in the data and the model is quite different, being (4).[5]

4 Case 3: More Co-integrating Vectors in the Data than the Model

In this instance the model has the VECM structure

$$\Delta z_t^M = \delta\gamma' z_{t-1}^M + \psi(z_{nt}^M - a_t) + e_t^M,$$

[5] When we have $\Delta z_t^D - \Delta z_t^M = \Delta v_t$ the solution for $z_t^D - z_t^M$ does not cumulate v_t.

while the VECM for the data has extra co-integrating vectors with the form

$$\Delta z_t^D = \delta\gamma' z_{t-1}^D + \alpha\beta' z_{t-1}^D + e_t^D. \tag{5}$$

Following the analysis of the previous section we would have

$$\eta_t = \Delta\xi_t = \delta\gamma'\xi_{t-1} + \alpha\beta' z_{t-1}^D - \psi(z_{nt}^M - a_t) + e_t^D - e_t^M. \tag{6}$$

Because $\alpha\beta' z_{t-1}^D - \psi(z_{nt}^M - a_t) + e_t^D - e_t^M$ is $I(0)$ then we will get co-integration again. To acheive that it is necessary to add the extra co-integrating vectors into the augmenting term. Notice that once again a choice of $\eta_t = \Delta v_t$ would effect co-integration, but clearly this is a mis-specification of the actual shock needed to reconcile data and model growth rates.

5 Conclusion

The note shows that working with the traditional form of errors in variables shocks in DSGE models would fail to produce co-integration between model variables and data, and would also imply that there is no co-integration between the data variables. If there is in fact co-integration in the data, and it has either the same or more co-integrating vectors as the model, then the traditional method of assuming that model and data growth rates differ by a white noise process results in a failure of model variables to co-integrate with the data. One can produce co-integration by working with differences in a white noise process, although the correct "errors in variables shocks" are far more complex, and involve a VARMA structure. Exactly what the consequences are of this mis-specification of the shock processes will be dependent upon the nature of the model. If one is happy to simply preserve co-integration between model and data variables using differences in white noise shocks, this would seem to be a relatively simple modification in programs that perform estimation with state space methods, such as Dynare.

6 References

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