The Impact of Thin Trading and Jumps on Realised Hedge Ratios*

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Abstract

The use of intra-daily data to produce daily variance measures has resulted in increased forecast accuracy and better hedging for many markets. However, this paper shows that improved hedging ratios can depend on the behaviour of price disruptions in the assets. When the spot and future prices for the same asset do not jump simultaneously within the day, then inferior hedging outcomes can be observed. We illustrate that this problem dominates bias from potential thin trading. Using US Treasury data we demonstrate how the extent of non-synchronised jumping, or disjoint jumping, leads to the finding that optimal hedging ratios are not improved with high frequency data in this market.

JEL Categories: C01, C32, G11, G17

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1 Introduction

A simple and widely used approach to reducing or eliminating exposure to price risk involves hedging spot positions with futures contracts. In a perfect hedge the profits and loss will offset exactly and the portfolio pays the riskless rate of return in equilibrium. The ratio of the number of units of the futures contracts held to relative to a unit of the spot contract is known as the hedge ratio. The optimal hedge ratio is that ratio which yields minium variance of the hedge portfolio.

Initial empirical research into hedge ratios considered the time invariant case as in Ederington (1979). This was followed by hedge ratios explicitly recognising the time varying joint distribution of cash and futures contracts; Kroner and Sultan (1993), and the application of GARCH techniques to capture time varying covariance such as Cechetti, Cumby and Figlewski (1988), Baillie and Myers (1991), Strong and Dickinson (1994), Park and Switzer (1995) and Brooks, Henry and Persand (2002), *inter alia*. However, Carnero, Peña and Ruiz (2004) argue that GARCH models fail to adequately capture the behaviours of squared returns, particularly the excess kurtosis and persistence typically observed in daily or even weekly data.

Recent focus has shifted to the potential of volatility measures constructed from intradaily data, specifically realized volatility, which has been shown to provide improved forecasts of future volatility in some instances.[1] The realized volatility approach dispenses with the need for an explicit model of the latent volatility process and instead provides a direct and consistent estimator of volatility. Lai and Sheu (2010) argue that realized measures of volatility based on intradaily data provide more accurate measures of the covariance matrix and hence produce better risk minimising hedges than can be obtain using daily data, and find that this leads to improved hedging outcomes with S&P500 data.

This study examines hedging price risk associated with the US Treasury bond market. In contrast with Lai and Sheu (2010), the use of intradaily data yields inferior dynamic hedges than obtained with daily data. In particular, the evidence suggests that the hedge ratios are bimodal, with one pronounced mode at zero. Two reasons may account for this finding. First, that trade is thin or asynchronous, so that finer intraday sampling increases the chance of no trading during that interval. The second is the presence and particular behaviour of price discontinuities, or jumps, in the two price series.

Jumps are known to be present in both US Treasury spot and futures data, see for

[1] Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold and Labys (2003), Barndorff-Nielsen and Shephard (2004), *inter alia*, use intradaily data to construct realized measures of volatility. Blair, Poon and Taylor (2001) and Andersen, Bollerslev, Christofferson and Diebold (2006), inter alia, investigate the use of realized volatility as a source of forecasts for future volatility.

example Johannes (2004), Anderson, Bollerslev and Diebold (2007), Dungey, McKenzie and Smith (2009), Jiang, Lo and Verdelhan (2010), while Piazzesi (2003) demonstrates improvements in yield curve modelling through accounting for potential jumps. Recently, Dungey and Hvozdyk (2010) have shown that while spot and futures prices in US Treasuries may jump contemporaneously, known as joint jumps, there is also a significant number of incidences when jumps in the two prices occur at statistically distinct times within a single day - these are known as disjoint jumps and occur in about 20% of jump cases.

We untangle the impact of non-syncrhonous or thin trading and jumps on estimated hedge ratios via a Monte Carlo experiment. The presence of thin trading creates some bias in the hedge ratio. However, this is dwarfed by the bias induced by the presence of jumps. Although joint jumps have relatively little impact, the occurrence of disjoint jumps creates a significant bias in the estimated hedge ratio, of up to 50% in some experiments. The results suggest that the potential for improved hedging using intradaily data depends critically on the extent of jump activity, and specifically disjoint jump activity; aspects which can be clearly identified using statistical tests proposed by Aït-Sahalia and Jacod (2010), Jacod and Todorov (2010) and Todorov and Tauchen (2010).

The results of this paper demonstrate that high frequency intra daily data will not always result in improved outcomes over daily data in producing daily measures of important financial management tools. In this case, although Lai and Shue (2000) show improvements in hedging with intraday equity data this turns out not to be the case for the US Treasury market. Given the importance of the US Treasury market in international portfolio management this is a striking result. The result derives from the recently documented extent of non-synchronised jumping in the spot and futures markets for this asset. Although the number of papers characterising the extent of joint and disjoint jumping in different assets is as yet relatively small, the US Treasury markets do seem to exhibit a greater degree of jumping than other markets, see for example Andersen et al (2007), and all types of jumps in this market are commonly associated with macroeconomic news; see Dungey and Hvozdyk (2011), Dungey, McKenzie and Smith (2009), Jiang, Lo and Verdelhan (2010). The disproportionate occurrence of disjoint jumps on news days means that hedging ratios constructed from intradaily data are most vulnerable on days when arguably they are most desired.

The paper is structured as follows. Section 2 presents the theoretical framework for deriving the hedge ratios using data at a daily frequency before moving to the construction of hedge portfolios using intradaily data. Section 3 describes the US Treasury data, and the hedging outcomes generated using daily and intradaily data. Section 4 presents Monte Carlo evidence on the role of thin trading and jump activity in producing these results and the practical implications. Finally, Section 5 concludes.

2 Optimal Hedging: An Empirical Analysis

Let $p_{s,t}$ and $p_{f,t}$ denote the price of the spot and futures contract, respectively at the end of day t. Similarly, let $r_{s,t}$ and $r_{f,t}$ denote the corresponding continuously compounded returns over day $t-1$ to t. The daily return to an unhedged position in the spot contract is therefore $r_{s,t}$. A hedge portfolio is constructed by simultaneously taking offsetting positions in the cash and futures markets. Denote the expected return to the hedge portfolio over the period $t-1$ to t as $E_{t-1}(r_{p,t})$:

$$E_{t-1}(r_{p,t}) = E_{t-1}(r_{s,t}) - \beta_{t-1} E_{t-1}(r_{f,t}). \tag{1}$$

Note that β_{t-1} represents the hedge ratio determined at time $t-1$, for employment in period t.[2] The conditional variance of the hedge portfolio, $\sigma^2_{p,t}$ is

$$\sigma^2_{p,t} = \sigma^2_{s,t} + \beta^2_{t-1}\sigma^2_{f,t} - 2\beta_{t-1}\sigma_{sf,t}, \tag{2}$$

where $\sigma^2_{s,t}$ and $\sigma^2_{f,t}$ are the conditional variances of the cash and futures contracts, respectively, and $\sigma_{sf,t}$ represents the corresponding conditional covariance. The optimal hedge ratio, β^*_{t-1}, for a risk averse utility maximising agent with mean-variance preferences is the value of β_{t-1} which minimises $\sigma^2_{p,t}$ given as

$$\beta^*_{t-1} = \frac{\sigma_{sf,t}}{\sigma^2_{f,t}}, \tag{3}$$

where β^*_{t-1} is unobserved and must be estimated appropriately.

2.1 Hedging Using Intradaily Data

Andersen, Bollerslev, Diebold and Wu (2005) introduce the notion of the realised measure of undiversifiable risk for a sample of equity returns. This paper build upon their approach to obtain a realised hedge ratio. Let the vector price process $p_t = (p_{s,t}, p_{f,t})$ follow a bivariate continuous-time stochastic volatility diffusion,

$$dp_t = \mu_t dt + \Omega_t dW_t, \tag{4}$$

where the standard 2-dimensional Brownian motion is denoted as W_t; the positive definite symmetric diffusion matrix, Ω_t, and the 2-dimensional instantaneous drift, μ_t, are strictly

[2] At this stage we do not require that β_{t-1} be time varying, but rather, require that it is determined using information available up to the end of period $t-1$.

stationary and are jointly independent of W_t. The covariance matrix contains the security variances $\sigma^2_{s,t} = \Omega_{(1,1)t}$ and $\sigma^2_{f,t} = \Omega_{(2,2)t}$ and the covariance $\sigma_{sf,t} = \Omega_{(1,2)t} = \Omega_{(2,1)t}$. The vector of continuously compounded returns is given by

$$r_{t+j\delta,\delta} = p_{t+j\delta,\delta} - p_{t+(j-1)\delta,\delta}, \tag{5}$$

where $1/\delta$ is the number of sampling intervals in the day. (It is useful to note in passing that $\delta = 1$ implies sampling at the daily frequency.) Conditioning on the realised sample paths of μ_t and Ω_t, it is possible to write the distribution of this return vector as

$$r_{t+\delta,\delta}|\sigma\left\{\mu_{t+\tau},\Omega_{t+\tau}\right\}_{\tau=o}^{1/\delta} \tilde{\ } N\left(\int_0^1 \mu_{t+\tau}d\tau, \int_0^1 \Omega_{t+\tau}d\tau\right), \tag{6}$$

where the σ-field generated by the sample paths of μ_t and Ω_t is written as $\sigma\left\{\mu_{t+\tau},\Omega_{t+\tau}\right\}$, for $0 \leq \tau \leq 1$. It follows that a natural measure of the true latent daily volatility is provided by the integrated diffusion matrix $\int_0^1 \Omega_{t+\tau}d\tau$.Under weak regularity conditions, Andersen, Bollerslev, Diebold and Wu (2005) show that

$$\sum_{j=1}^{1/\delta} r_{t+j\delta,\delta}r'_{t+j\delta,\delta} - \int_0^1 \Omega_{t+\tau}d\tau \longrightarrow 0 \tag{7}$$

almost surely for all t as the return sampling frequency increases, $\delta \longrightarrow 0$. In order to construct the realised hedge ratio it is necessary to obtain the appropriate elements of the realized diffusion matrix, namely the realised variance of the futures contract

$$RV_{f,t+1} = \sum_{j=1}^{1/\delta} r^2_{f,t+j\delta,\delta}, \tag{8}$$

and the realised covariance

$$RV_{sf,t+1} = \sum_{j=1}^{1/\delta} r_{f,t+j\delta,\delta}r_{s,t+j\delta,\delta}. \tag{9}$$

The realised hedge ratio is the ratio of (9) to (8), and it follows from the theory of quadratic variation, under fairly weak regularity conditions, that

$$\hat{\beta}_{R,t+1,\delta} = \frac{RV_{sf,t+1}}{RV_{f,t+1}} \longrightarrow \beta^*_{t+1} \equiv \frac{\int_0^1 \Omega_{(1,2)t}}{\int_0^1 \Omega_{(2,2)t}}, \tag{10}$$

almost surely for all t as the return sampling frequency increases, $\delta \longrightarrow 0$, ensuring that the realised hedge ratio is consistent for the corresponding integrated hedge ratio.

3 Hedging Outcomes for US Treasuries

3.1 Data

Data on the 5 year maturity US Treasury bond spot transactions are obtained from Cantor Fitzgerald with corresponding futures markets observations from the Chicago Mercantile Exchange. The raw data comprise tick-by-tick transaction prices for the period 2 January 2002 to 29 December 2006. After excluding weekends and public holidays the sample includes 1230 trading days[3]. The closing price is selected as that closest to 5:30 PM EST each day, and daily returns calculated according to (5) with $\delta = 1$, and appropriately scaled to yield daily percentage returns. Summary statistics for the daily returns data are presented in Panel A of Table 1. As is usual with asset returns the raw return data are approximately mean zero and displays evidence of non-normality and heteroskedasticity. A test for up to 5th order ARCH is strongly significant for both $r_{s,t}$ and $r_{f,t}$.

3.2 Daily Measures of the Hedge Ratio

Table 1 B reports the unhedged variance of the spot contracts in the first column, with the first row giving the in-sample variance for the first 1000 observations, and the second the out-of sample variance for the case where the final 230 observations are used as an out-of-sample case. The remaining columns report the portfolio return variance under alternative hedging strategies. The second column, headed $\beta = 1$ refers to the naive strategy of offsetting every cash contract with a futures contract, which assumes both constant conditional variance-covariance matrix of returns, and that $\sigma_{sf} = \sigma_f^2$ holds. In this case there is a 0.125% reduction in in-sample variance over the no-hedging outcome of column 1, but in the out-of sample case, there is a 10.005% deterioration in performance.

In more sophisticated hedging strategies the time series behaviour of Σ_t, the conditional variance-covariance matrix of returns, will determine the appropriate approach to estimating β^*_{t-1}. Consider the constant variance-covariance matrix Σ:

$$\Sigma = \begin{bmatrix} \sigma_s^2 & \sigma_{sf} \\ \sigma_{sf} & \sigma_f^2 \end{bmatrix}. \tag{11}$$

In this case, the optimal hedge ratio will also be constant (see Ederington, 1979 *inter alia*) and its best estimate may be obtained as the OLS slope coefficient from the regression

$$r_{s,t} = \mu + \hat{\beta} r_{f,t} + u_t. \tag{12}$$

OLS estimates of the optimal hedge ratio are reported in column 3 of Panel B in Table 1, headed *OLS*. Using the initial 1000 observations of $r_{s,t}$ and $r_{f,t}$ the OLS hedge ratio

[3] We define the trading day as starting at 7:30 AM and finishing at 5:30 PM (EST) time.

$\hat{\beta}$ is calculated and is used to calculate $r_{p,t}$ using (1) and hence $Var(r_{p,t})$. In comparison to the unhedged case, $\beta = 0$, the OLS approach yields an in-sample gain to hedging of 33.68%. To construct the out-of-sample OLS hedge ratio estimates the sample window is extended to encompass the remaining 230 observations; the corresponding $r_{p,t}$ and its variance are calculated for each of the 230 days. The results are displayed in the out-of sample outcomes for the $\hat{\beta}$ estimates in column 4 of Panel B in Table 1. In this out-of-sample case, the improvement from hedging is 45.031% over the unhedged case.

However, it is unlikely that the conditional variance-covariance matrix is actually constant, in which case the optimal hedge ratio will display time variation and is unlikely to be unity at any time t. Abstracting from rebalancing cost of the hedge portfolio, a constant hedge ratio would yield inferior hedging outcomes when there is time variation in the elements of Σ_t. Let $\hat{\sigma}^2_{s,t}$ and $\hat{\sigma}^2_{f,t}$ and $\hat{\sigma}_{sf,t}$ represent the elements of the estimated conditional covariance matrix, $\hat{\Sigma}_t$, where

$$\hat{\Sigma}_t = \begin{bmatrix} \hat{\sigma}^2_{s,t} & \hat{\sigma}_{sf,t} \\ \hat{\sigma}_{sf,t} & \hat{\sigma}^2_{f,t} \end{bmatrix}. \tag{13}$$

The optimal hedge ratio may be estimated as

$$\hat{\beta}_{t-1} = \hat{\sigma}_{sf,t} / \hat{\sigma}^2_{f,t}. \tag{14}$$

GARCH models may be used to obtain the elements of the estimated conditional covariance matrix and hence $\hat{\beta}_{G,t-1}$. Defining $r_t = (r_{s,t}, r_{f,t})'$ and $u_t = (u_{s,t}, u_{f,t})'$, we employ quasi-maximum likelihood techniques to estimate the following system of equations

$$\begin{array}{c} r_t = \Phi_0 + \sum_{k=1}^{2} \Phi_k r_{t-k} + u_t \\ \Sigma_t = C_0^{*\prime} C_0^* + A_{11}^{*\prime} u_{t-1} u_{t-1}' A_{11}^* + B_{11}^{*\prime} \Sigma_{t-1} B_{11}^* \end{array}, \tag{15}$$

where Φ_0 is a 2×1 matrix of intercepts, C_0^* is a 2×2 coefficient matrix, which is restricted to be upper triangular to ensure that (15) is identified, and Φ_k, A_{11}^*, B_{11}^* are 2×2 coefficient matrices. This parameterisation of the bivariate GARCH system, due to Engle and Kroner (1995), ensures that Σ_t is positive-definite for all values of u_t in the system. The time varying hedge ratio may be obtained from $\hat{\Sigma}_t$ using (14) and used to construct the hedge portfolio return $r_{p,t}$.

The column headed $GARCH$ in panel B of Table 1 reports the portfolio variance for the in-sample and out-of-sample cases where the GARCH specification of (15) is applied to the initial 1000 observations for in-sample and one-step ahead estimators used to calculate the 230 out-of-sample days and the associated $r_{p,t}$ and the associated variance. The in-sample variance is reduced by 33.771% over the no-hedging case while the out-of-sample variance is reduced by 43.402%.

It is clear from Table 1 that any hedge, other than a naive hedge, leads to a reduction in the exposure to risk within sample, that is, $Var(r_{p,t}) < Var(r_{s,t})$ for all portfolios considered. The out of sample results suggest that the naive hedge actually leads to an increased risk with the variance of the naive hedge portfolio actually increasing. However, the OLS and GARCH approaches to hedging yield substantial reductions in volatility. The evidence in Table 1 suggests that there is little to choose between the OLS and GARCH approaches. This is somewhat at odds with the general consensus in the recent literature for other assets that the use of multivariate GARCH models yields superior minimum variance hedges than the fixed-sample or rolling-sample OLS approach, see Cecchetti et al. (1988), Baillie and Myers (1991) and Brooks et al. (2002) *inter alia.*

3.3 Intradaily data

We construct equally spaced prices for the spot and futures contracts by setting the price in the j^{th} interval of day t to be equal to the last trading price recorded in that interval. Continuously compounded returns for each interval are calculated as shown in (5). For ease of interpretation the difference of the log transaction price is scaled to yield percentage returns for each sampling frequency δ. For instance, with the 5-minute sampling frequency the 10-hour trading day is partitioned into 120 intervals so that $\delta = 1/120$. There is no uniform agreement over the choice of sampling frequency in the high-frequency data. A number of sampling frequencies have been chosen in the empirical papers; for example, Lahaye et al. (2009) choose 15 minute intervals, while Huang and Tauchen (2005) and Andersen et al. (2007) sample their data at 5 minute intervals. Ongoing research addresses the optimal univariate sampling case, such as Bandi and Russell (2005) but the problem is more difficult for multiple series with possibly different trading intensities. We construct three discrete and equally spaced intradaily sampling frequencies, namely 5, 30 and 60-minute intervals, and calculate the realised variance and realised covariance and hence realised hedge ratio for each frequency. We may then construct hedges for the following day using

$$r_{p,t} = r_{s,t} - \hat{\beta}_{R,t-1,\delta} r_{s,t} \tag{16}$$

for alternative sampling frequencies, obtaining various hedge portfolio returns and corresponding variance estimates. The intradaily in-sample and out-of-sample variances are reported in panel C of Table 1 for each frequency. The first column of panel C repeats the benchmark unhedged position based on daily data. The three remaining columns report the cases using 5 minute sampling, 30 minute sampling and 60 minute sampling. In both the in-sample and out-of-sample cases the hedges based on 5 minute sampling frequency produce the largest reduction in exposure compared with the unhedged benchmark. The largest effects are available in the out-of-sample cases, where the reduction in variance is uniformly over 23% and is largest for the 5 minute sampling frequency which achieves a 27.277% reduction in exposure. The largest in-sample gain is 16.703% in the 5 minute case, however the in-sample performance at the 30 and 60 minute sampling frequencies are inferior, with a reduction risk of 3.656% and 5.341%. At the 120 minute sampling frequency there is a reduction of 20.668% in exposure to price risk.

Comparing panels B and C of Table 1 makes clear that the intradaily hedging outcomes are dominated by the daily OLS and GARCH approaches to hedging, whether one examines the in-sample or out-of-sample evidence. This is at odds with recent evidence of the effectiveness of hedging strategies which employ intradaily data to hedge foreign exchange rate exposures, Harris, Shen and Stoja (2010), or equity risks, Lai and Sheu (2010). The remainder of this paper seeks to explain the relatively poor performance of the hedges based on $\hat{\beta}_{R,t-1,\delta}$.

3.4 Intradaily Measures of the Hedge Ratio

Table 2 presents summary statistics for the various realized hedge ratios, $\hat{\beta}_{R,t-1,\delta}$ for 5, 30 and 60-minute sampling frequencies, and for the GARCH hedge ratio $\hat{\beta}_{G,t}$ using the in-sample data. The average of the $\hat{\beta}_{R,t-1,\delta}$ appears to increase with sampling frequency, δ. The average value of $\hat{\beta}_{G,t}$ is less than that of $\hat{\beta}_{R,t-1,\delta}$ for the 30 and 60-minute sampling frequencies. The variance of $\hat{\beta}_{R,t-1,\delta}$ appears to peak at the 30-minute sampling frequency and then decline. In every case the variance of the realized beta exceeds that of the hedge ratio obtained using the GARCH approach.

All approaches to obtaining the hedge ratio appear to be non-normal in distribution. For the hedge ratios constructed with intradaily data, the skewness becomes increasingly negative as the sampling frequency declines. Moreover, the distribution of $\hat{\beta}_{R,t-1,\delta}$ appears platykurtic, although the extent of the platykurtosis declines with the sampling frequency. In contrast, the GARCH hedge ratio although negatively skewed displays a large degree of excess kurtosis.

Figure 1 displays the estimated unconditional distributions of $\hat{\beta}_{R,t-1,\delta}$ for the various sampling frequencies. All the distributions appear bimodal with a pronounced mode at zero and a second mode in the interval $[0.5, 1]$. For the 5-minute sampling frequency the dominant mode is at zero, while for the 30, 60 and 120-minute sampling frequencies the dominant modes are closer to unity.

Recall that the realized beta is defined as $\hat{\beta}_{R,t-1,\delta} = RV_{sf,t-1}/RV_{f,t-1}$. Three particular circumstances may lead to a downward bias in $\hat{\beta}_{R,t-1,\delta}$ from its true value for day t. In the first case, $r_{s,t,\delta} = 0$, leading to a downward bias in $RV_{sf,t-1}$ across the day. In the second case, $r_{f,t,\delta} = 0$ and consequently both $RV_{sf,t-1}$ and $RV_{f,t-1}$ will be biased downwards. In the third case both $r_{s,t,\delta} = 0$ and $r_{f,t,\delta} = 0$ and again both $RV_{sf,t-1}$ and $RV_{f,t-1}$ are biased. Where any of these three scenarios occur in any interval within the day the realized beta for that day will be biased downwards and the magnitude of the bias increases as the number of intervals containing zero increases.

This paper explores two possible candidate explanations to the large number of zero or low outcomes for $\hat{\beta}_{R,t-1,\delta}$. In the first case no trading occurs in the cash and/or futures market in interval δ, and so $r_{s,t,\delta} = 0$ and/or $r_{f,t,\delta} = 0$. This corresponds to the Epps (1979) effect where thin trading results in a downward bias to the hedge ratio. Closer examination of Figure 1 suggests that the mode at 0 reduces as the sampling frequency declines. Moreover, the mode located in the interval $[0.5, 1]$ increases in size as the sampling frequency declines, consistent with thin trading as the chance of a trade occurring within any intradaily interval increases as the interval length increases. However, increasing the interval length necessarily reduces the number of intradaily observations available.

In the second case while trade may or may not occur in a particular intradaily interval, a jump occurs in the interval causing $\hat{\beta}_{R,t-1,\delta}$ to deviate from its true underlying continuous value resulting from either a deviation of either the covariance, the realized futures variance or both.

The jump behavior of US Treasuries has now been documented in a number of places. Univariate jump tests such as those proposed by Barndorff-Nielsen and Shephard (2004) and variations on Lee and Mykland (2008) are reported in Andersen et al (2007), Dungey, McKenzie and Smith (2009), Jiang, Lo and Vedelhan (2010). An interesting feature of this market is the relatively higher proportion of days exhibiting jump behavior than in the more frequently investigated equity market data - see for example the comparison in Andersen et al (2007). This may be part of the explanation of why Lai and Sheu (2010) find improvements for hedging outcomes using intradaily data for an equity index compared with the individual maturity bonds investigated here. To illustrate the extent of jumps in the data, Table (3) reports the rejection frequency of the no jump hypothesis for univariate spot and futures data for different maturities of bond using the Barndorff-Nielsen and Shephard (2004) test. It is clear that futures prices jump more frequently than spot, and that the rejection frequency decreases with maturity, although the short end and longest maturities jump most frequently, consistent with information arrival at both ends of the maturity structure. In the hedging application we are concerned with the impact of jumps in two assets, futures and spot. Several scenarios may arise on any given day; where neither asset jumps, where one asset jumps and the other does not, where both assets jump contemporaneously and where both assets jump in the day but at statistically distinct times. However, a recent test by Jacod and Todorov (2010) builds on the Barndorff-Nielsen and Shephard (2004) univariate results to distinguish statistically contemporaneous jumps, known as joint jumps, from those which are statistically separable in time, known as disjoint jumps. Their proposal consists of two tests. The first, the joint jump test, uses the null of no contemporaneous jump and tests whether the ratio of realized variances constructed using different sampling frequencies is the same, in which case the jumps occurred statistically simultaneously. The second test, the disjoint jumps test, operates under a null that jumps observed in the same day are not contemporaneous, and tests whether the ratio of the realized variance to the square of the product of the quadratic returns of the individual assets is statistically different to zero. In the case that this ratio is zero, then the asset returns do not move together.

Dungey and Hvozdyk (2011) apply this test to the US Treasury spot and futures. The corresponding results for the data used in this paper are reported in Table 4 for 5 minute sampling, and clearly show that both joint and disjoint jumps are common occurrences in the US Treasury data. There is very little comparative work on these cojumping tests in the literature as yet, applications are limited to a December 1986 to June 1999 sample of DM/USD and JPY/USD data in Jacod and Todorov (2009) and a comparison of price and volatility jumps in the S&P500 futures for January 1997 to June 2007. Our sample reports higher incidence of both cojumping and disjoint jumping than either of the exchange rate or equity data studies, although this must be regarded as relatively early evidence given the disparity in sample periods and sampling frequency of the different studies.

To investigate the relative effects of thin trading and jumps the next section employs a Monte Carlo experiment which allows us to vary the degrees of thin trading and the type, frequency and size of jumps and whether the jumps occur contemporaneously or in a disjoint manner.

4 The Monte Carlo Experiment

4.1 Data Generating Process

The MC experiment creates of 1000 (days) realizations of the pairs of simulated return series. We simulate 1 increment per 5 minutes for a trading day of 10 hours. The dynamics of the price series p_i, where i indexes the assets in the experiment, is a version of the stochastic volatility jump diffusion processes of Andersen et al. (2007) and Veraart (2010):

$$\begin{array}{l} dp_{i,t} = \exp(\zeta_0 + \zeta_i v_{i,t}) dW_{i,t} + \alpha_i \int_{\mathbb{R}} p_i \lambda_i(dt, dp_i) + \alpha_{sf} \int_{\mathbb{R}} p_i \lambda_3(dt, dp_i) \\ dv_{i,t} = \alpha_v v_{i,t} dt + dW_t^{v_i},\ i = \{s, f\}, \end{array} \quad (17)$$

where ζ_0 and ζ_i govern the drift and trend component of the continuous price process, W_t is a standard Brownian motion. The stochastic volatility component is $v_{i,t}$ with magnitude determined by α_v, while the correlation between the return of security i and v_i is governed by $cor(W_i, W^{v_i}) = \rho_i$.

Equation (17) allows for jumps as sudden discontinuities in the continuous price process. The parameters λ_1, λ_2 and λ_3 are Poisson measures that govern the intensity of the jumps, while the magnitudes of the jumps are determined by the loadings α_s, α_f and α_{sf}. Following Jacod and Todorov (2009) and Dungey and Hvozdyk (2011) a joint jump occurs where $\alpha_s = \alpha_f = 0$ and $\alpha_{sf} \neq 0$, that is a jump occurs contemporaneously within a day. A disjoint jump occurs where $\alpha_s = \alpha_f \neq 0$ and $\alpha_{sf} = 0$, in which case the two series may jump at different time intervals within a day. In the absence of jumps, $\alpha_s = \alpha_f = \alpha_{sf} = 0$, while when both types of jumps are present $\alpha_s = \alpha_f = \alpha_{sf} \neq 0$.

Thin trading occurs where there is a periodic lack of volume arriving to the market. If the return r_i is zero in an interval on day t due to thin trading, then the quotient $\hat{\beta}_{R,t-1,\delta} = RV_{sf,t-1}/RV_{f,t-1}$ will be biased downwards relative to the true value. In the MC experiments to follow the degree of thin trading is determined by the parameter ϕ, the proportion of intervals in a trading day which contain zero return. Given randomly generated data and assuming that thin trading is uniformly distributed across the day, it is possible to impose zero values on the proportion ϕ of the returns after the data is generated. When $\phi = 0$, there is an absence of thin trading, and the prevalence of thin trading increases with ϕ up to 20% of intradaily intervals (well beyond the less than 5% of intervals with no trade in the 5 minute Treasury database). This will result in an excess of zero return observations as the price in the market does not change as frequently as would occur under more usual trading conditions. We also allow for the possibility that thin trading occurs independently in the cash and futures markets.

The parameter values chosen follow Jacod and Todorov (2009) and set $\alpha_v = 0.1$, $\zeta_0 = 0$, $\zeta_i = 0.125$, and in line with Veraart (2010) $\rho_i = -0.62$. These values are fixed for all experiments. We draw the individual asset price data from the standard normal distribution, implying that the true value of beta will be determined by the correlation between the assets. This correlation is initially chosen to be 1.0 but is varied for completeness to allow for the situations where no perfect hedge exists, such as the use of a Treasury bond future to hedge the risk of a corporate bond position.

We consider four scenarios regarding jumps, namely no jumps, joint and disjoint jumps, joint jumps only, and disjoint jumps only, with no thin trading effects. Further, we consider the effect of varying the degree of thin trading from $\phi = 0$ to $\phi = 0.2$ in the absence of jumps[4]. Finally, by allowing for both thin trading and jumps we are able to investigate the relative impact of each factor.

In all experiments we simulate 10 hour trading day with 120 pseudo 5-minute intervals. This yields 120,000 intradaily intervals which we allow to be randomly affected by 5000 common jumps and/or 5000 disjoint jumps which are assumed to uniformly distributed across the whole returns vector. Sub-sampling these pseudo 5-minute data appropriately yields 20 pseudo 30-minute, 10 pseudo 60-minute and 5 pseudo 120-minute returns per day. We measure the sensitivity of our results by varying both the magnitude of the jump components, the relative jump intensities, the correlation across assets, and the sampling frequency. Two correlation values are examined, the case where the true value of $\beta_{R,t-1,\delta}$ is unity by construction as $var(r_s) = var(r_f) = 1.0$ and $corr(r_s, r_f) = 1.0$ with results reported in Tables (6) to (8) and the case where the true value of $\beta_{R,t-1,\delta} = 0.5$ in Table (9).

4.2 Results

We first examine the results of thin trading in the absence of jumps for different sampling frequency, reported in Panel A of Table (6). In the absence of thin trading, $\phi = 0.0$, all approaches yield unbiased estimates of $\beta_{R,t-1,\delta}$. Holding the sampling frequency constant, the impact of thin trading results in no change in estimator uncertainty alone at the pseudo 5-minute frequency. At the pseudo 30- (60-) [120-] minute sampling frequency the estimate of $\beta_{R,t-1,\delta}$ is biased upward by up to 0.4% (0.9%) [1.8%], while the uncertainty of the estimate changes from 0 to 0.103 (0.161) [0.265] as the degree of thin trading increases.

Panel B of Table (6) considers the impact of jumps. In the absence of thin trading, the presence of jumps alone, without distinguishing between joint and disjoint jumps, leads to a downward bias in the estimate of $\beta_{R,t-1,\delta}$ of $8.8\% - 8.9\%$. At the pseudo 5-minute sampling frequency the only impact of thin trading is a very marginal increase in estimation uncertainty. At the lower sampling frequencies, both the estimate of $\beta_{R,t-1,\delta}$ and its standard deviation increase fractionally as the degree of thin trading rises. Taken together, the results in Panels A and B of Table (6) demonstrate that the estimate of $\beta_{R,t-1,\delta}$ is biased downwards by approximately 9% by the presence of jumps in the data irrespective of the sampling frequency or the degree of thin trading.

To determine whether the type of jumps matters in determining this bias we proceed to decompose the jump into joint and disjoint processes. Panel C of Table (6) reports

[4]In our data there are up to 4% of intradaily intervals containing no trade records on average in the 5 year maturity at the 5-minute sampling frequency.

the results where only joint jumps occur. Here the series jump simultaneously and the magnitude of the jumps is scaled by $\alpha_s = \alpha_f = 0.6$. These joint jumps appear to increase the correlation between $r_{s,t}$ and $r_{f,t}$ leading to very accurate estimates of $\beta_{R,t-1,\delta}$ and very little estimation uncertainty at the pseudo 5-minute frequency, with thin trading having no discernible impact irrespective of the value of ϕ. At the lower sampling frequencies the upward bias in the standard deviation of the realized hedge ratio is increasing in ϕ.

The impact of disjoint jumps of a similar magnitude (governed by $\alpha_{sf} = 0.6$) are reported in Panel D of Table (6). Their effect on the mean and standard deviation of the estimate of $\beta_{R,t-1,\delta}$ is marked. The expected value of the realized hedge ratio is biased downwards by approximately 30%, irrespective of the magnitude of ϕ. The main impact of thin trading is to inflate the uncertainty of the estimate, with the size of the bias being increasing in ϕ.

Tables (7) to (9) present sensitivity analysis to the original experiment reported in Table (6). The results reported in Table (7) reduce the magnitude of the jumps by 50%; that is $\alpha_s = \alpha_f = \alpha_{s,f} = 0.3$. Other than very marginal changes in magnitude the outcomes are quantitatively and qualitatively robust to this change. In Table (8) represent the case where the ratio of joint to disjoint jumps is increased to 10:1. Panels B and D of this table demonstrate that having fewer disjoint jumps reduces bias to approximately 4.5% and 15%, respectively (in contrast to 9% and 30% in the case when there are more disjoint jumps, see Table (6)). In Table 6 the two series were perfectly correlated. Reducing the correlation between the returns series to 0.5 and keeping both joint and disjoint jumps leads to an upward bias in the estimate of $\beta_{R,t-1,\delta}$ by up to 80% as reported in Panel B of Table (9), and up to 96% when only joint jumps are present in Panel C. The results in Panel D indicate that disjoint jumps alone create downward bias in the estimates of $\beta_{R,t-1,\delta}$ of about $20\% - 30\%$, not dissimilar to the original results.

The implications of our results for agents hedging risk is clear. Where assets jump in a random fashion some caution should be exercised in establishing hedge positions based on intradaily data. The signal in the intradaily data appears to be heavily impacted by price discontinuities or jumps, and most particularly this is the case when the jumps are not contemporaneous. Furthermore, while thin trading has some effects, these must be considered as second order in comparison with the impact of jumps. In short, while intradaily data presents an opportunity to establish estimates of the hedge ratio based on the very large sample sizes, this opportunity is not costless. It may well be that the hedges based on the smoother daily data outperform their intradaily counterparts. This appears consistent with the evidence we obtain using our sample of US Treasury bonds and futures.

5 Conclusion

Unless the conditional variance-covariance matrix of spot and futures returns is time invariant, any optimal hedge ratio must display time variation. There is a deep literature discussing estimation of time varying hedge ratios for dynamic hedges at the daily frequency (see Ederington, 1979; Baillie and Myers, 1993; and Brooks, Henry and Persand, 2002 *inter alia*). Harris et al. (2010) employ intradaily data on foreign exchange contracts, while Lai and Sheu (2010) use intradaily equity security data to obtain realized measures of variance and covariance and hence to construct the realized hedge ratio. Using these intradaily estimates Harris et al. (2010) and Lai and Sheu (2010) obtain superior in-sample and out-of-sample hedges relative hedges based on daily measures.

In contrast, this paper finds that the outcomes of hedging US Treasury bond risk using intradaily data yields inferior results relative to risk minimization strategies based on daily data. The unconditional distribution of the realized hedge ratios is bimodal and contains an unusually large number of zero realizations. It is well known that the US Treasury bond and futures contracts display jump dynamics in the intradaily domain. Furthermore, there is the possibility that the data displays thin trading where trading does not occur in every intradaily interval. If either the cash or futures contract jumps or is thinly traded, we conjecture that the realized hedge ratio will be biased towards zero.

A series of Monte Carlo experiments are employed to examine whether thin trading and jumps actually lead to a biased estimate of the hedge ratio. We find that thin trading alone leads to upward biases in the order of 2%. In the presence of jumps there is a bias downwards of the hedge ratio of approximately 9%. Neither the coefficient bias nor the associated measure of uncertainty are manifestly affected by increasing degrees of thin trading. Decomposing the jump component in our DGP into joint and disjoint jumps we find that joint jumps have a minimal impact on the coefficient estimate and its associated standard deviation. In contrast, the impact of disjoint jumps results in downward bias in the estimated hedge ratio of around 30%. This bias is unaffected by increasing the degree of thin trading. Thin trading does however inflate the bias in the standard deviation of the hedge ratio. In the most severe scenario where one interval in five contains no trade, the resulting bias in the realized hedge ratio is swamped by the effect of the jumps we consider. This result is robust to variations in the magnitude of the jumps. When correlation between two series decreases, the upward bias of the joint jumps and the downward bias of the disjoint jumps are severe and are approximately 96% and 20%-30%, respectively.

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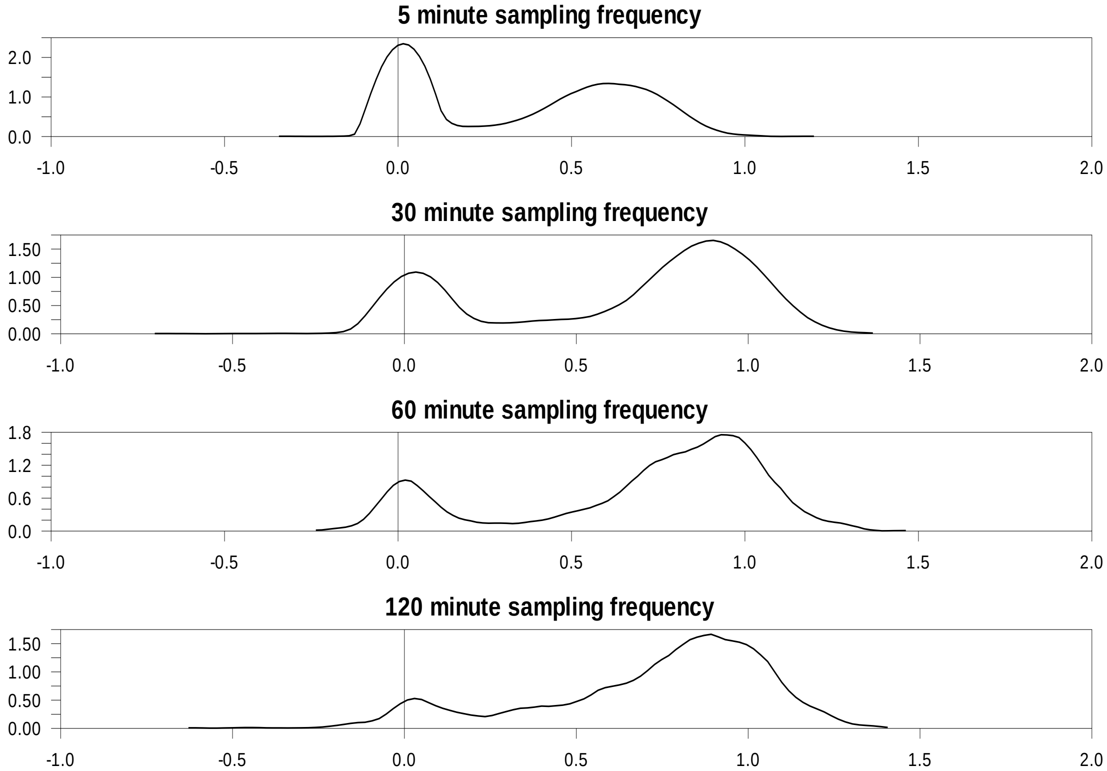

Figure 1: Bimodality of Realised Beta

Table 1: Hedging Outcomes: Empirical Hedging Exercise

Panel A: Summary Statistics

	$Mean$	Var	SK	EK	JB	$Q(5)$
r_s	-0.0001	0.1371	0.0004	0.0000	0.0004	0.0000
r_f	0.0027	0.1842	0.0000	0.0000	0.0000	0.0041

Panel B: Hedging Outcomes: Daily Data

	No Hedge	Naive Hedge	OLS Hedge	GARCH Hedge
In-Sample Variance	0.1371	0.1369	0.0909	0.0908
% reduction	-	0.125	33.683	33.778
Out-Sample Variance	0.033	0.0360	0.0179	0.0184
% reduction	-	-10.005	45.031	43.402

Panel B: Hedging Outcomes: Intradaily Data

	5-Minute Frequency	30-Minute Frequency	60-Minute Frequency	120-Minute Fequency
In-Sample Variance	0.114	0.132	0.130	0.115
% reduction	16.697	3.656	5.341	15.762
Out-Sample Variance	0.024	0.025	0.024	0.026
% reduction	27.277	23.887	26.434	20.669

Table 2: Descriptive Statistics for the Time Varying Hedge Ratios

	Mean	**Var**	**Skew**	**Ex. Kurt**	**J-B**
5-Minute Frequency	0.352	0.096	0.050	-1.547	100.098
			(0.368)	(0.000)	(0.000)
30-Minute Frequency	0.623	0.156	-0.633	-1.061	111.215
			(0.000)	(0.000)	(0.000)
60-Minute Frequency	0.696	0.137	-0.846	-0.461	127.641
			(0.000)	(0.000)	(0.000)
120-Minute Frequency	0.720	0.116	-0.929	0.262	146.259
			(0.000)	(0.092)	(0.000)
GARCH	0.528	0.027	-0.637	11.084	5166.172
			(0.000)	(0.000)	(0.000)

Table 3: Barndorff-Neilsen and Shephard (2004) univariate jump tests for the US Treasury bond and futures sampled at 5 minute intervals, January 2002 – December 2006

	BOND		FUTURES	
Maturity	No. of jump days	Rejection frequency	No. of jump days	Rejection frequency
2 Year	916	0.745	1104	0.898
5 Year	451	0.367	728	0.592
10 Year	484	0.393	652	0.530
30 Year	574	0.467	586	0.476

Table 4: Jacod and Todorov (2010) cojumping tests for spot-future maturity pairs of US Treasuries at 5 minute sampling, January 2002 – December 2006

Maturity	Joint Jumps, $\Phi_t^{(j)}$		Disjoint Jumps, $\Phi_t^{(d)}$	
	Number of days	Rejection frequency	Number of days	Rejection frequency
2 year	34	0.028	708	0.576
5 year	77	0.063	243	0.198
10 year	158	0.128	226	0.184
30 year	241	0.196	246	0.200

Table 5: Parameter Settings for the Monte Carlo Simulation

ρ	ϕ	$\alpha_1 = \alpha_2 = \alpha_3$	ρ	ϕ	$\alpha_1 = \alpha_2$	α_3
1.00	0.00	0.0	1.00	0.00	0.0	0.6
1.00	0.00	0.6	1.00	0.00	0.6	0.0
1.00	0.05	0.0	1.00	0.05	0.0	0.6
1.00	0.05	0.6	1.00	0.05	0.6	0.0
1.00	0.10	0.0	1.00	0.10	0.0	0.6
1.00	0.10	0.6	1.00	0.10	0.6	0.0
1.00	0.15	0.0	1.00	0.15	0.0	0.6
1.00	0.15	0.6	1.00	0.15	0.6	0.0
1.00	0.20	0.0	1.00	0.20	0.0	0.6
1.00	0.20	0.6	1.00	0.20	0.6	0.6

Table 6: Mean and Standard Deviation of Beta Across Sampling Frequencies, 1000 Replications Over 1000 Days, DGP With or Without Jumps, Ratio of Joint to Disjoint Jumps is 5:1

Sampling Frequency	**5 min**	**30 min**	**60 min**	**120 min**	**5 min**	**30 min**	**60 min**	**120 min**
	*A. Without Jumps**				*B. With Jumps***			
	$\phi = 0.00$							
Beta	1.000	1.000	1.000	1.000	0.911	0.911	0.911	0.912
St dev	0.000	0.000	0.000	0.000	0.197	0.244	0.299	0.421
	$\phi = 0.05$							
Beta	1.000	1.001	1.002	1.005	0.911	0.910	0.910	0.911
St dev	0.000	0.050	0.077	0.125	0.201	0.271	0.349	0.515
	$\phi = 0.10$							
Beta	1.000	1.002	1.005	1.010	0.911	0.911	0.911	0.910
St dev	0.000	0.071	0.110	0.181	0.206	0.302	0.404	0.609
	$\phi = 0.15$							
Beta	1.000	1.003	1.007	1.014	0.911	0.912	0.912	0.912
St dev	0.000	0.088	0.138	0.225	0.211	0.337	0.463	0.708
	$\phi = 0.20$							
Beta	1.000	1.004	1.009	1.018	0.912	0.912	0.913	0.913
St dev	0.000	0.103	0.161	0.265	0.216	0.380	0.533	0.831
	*C. Joint Jumps Only****				*D. Disjoint Jumps Only*****			
	$\phi = 0.00$							
Beta	1.000	1.000	1.000	1.000	0.670	0.670	0.670	0.668
St dev	0.000	0.000	0.000	0.000	0.535	0.891	1.232	1.872
	$\phi = 0.05$							
Beta	1.000	1.000	1.000	1.000	0.681	0.681	0.681	0.679
St dev	0.000	0.111	0.175	0.288	0.539	0.914	1.266	1.933
	$\phi = 0.10$							
Beta	1.000	1.000	1.000	1.000	0.693	0.694	0.693	0.694
St dev	0.000	0.171	0.267	0.446	0.544	0.942	1.315	2.021
	$\phi = 0.15$							
Beta	1.000	1.000	1.000	1.001	0.703	0.704	0.706	0.711
St dev	0.000	0.228	0.353	0.572	0.546	0.965	1.347	2.096
	$\phi = 0.20$							
Beta	1.000	1.000	1.001	1.001	0.715	0.717	0.719	0.723
St dev	0.000	0.280	0.437	0.719	0.550	0.990	1.391	2.162

Note: ϕ is the asynchronicity parameter of futures. *Jump size parameters of equation (17) are $\alpha_1 = \alpha_2 = \alpha_3 = 0$. **Jump size parameters of equation (17) are$\alpha_1 = \alpha_2 = \alpha_3 = 0.6$. ***Jump size parameters of equation (17) are $\alpha_3 = 0.6$ and $\alpha_1 = \alpha_2 = 0$. ****Jump size parameters of equation (17) are $\alpha_3 = 0$ and $\alpha_1 = \alpha_2 = 0.6$. 5000 joint jumps, 1000 disjoint jumps (500 disjoint jumps in futures, 500 disjoint jumps in spot).

Table 7: Mean and Standard Deviation of Beta Across Sampling Frequencies, 1000 Replications Over 1000 Days, DGP With or Without Jumps, Jump Size is Reduced by 50 Percent

Sampling Frequency	**5 min**	**30 min**	**60 min**	**120 min**	**5 min**	**30 min**	**60 min**	**120 min**
	*A. Without Jumps**				*B. With Jumps***			
				$\phi = 0.00$				
Beta	1.000	1.000	1.000	1.000	0.914	0.914	0.914	0.915
St dev	0.000	0.000	0.000	0.000	0.187	0.221	0.261	0.354
				$\phi = 0.05$				
Beta	1.000	1.001	1.002	1.005	0.914	0.914	0.914	0.914
St dev	0.000	0.050	0.077	0.125	0.190	0.240	0.296	0.420
				$\phi = 0.10$				
Beta	1.000	1.002	1.005	1.010	0.914	0.915	0.915	0.914
St dev	0.000	0.071	0.110	0.181	0.194	0.259	0.331	0.481
				$\phi = 0.15$				
Beta	1.000	1.003	1.007	1.014	0.915	0.915	0.915	0.916
St dev	0.000	0.088	0.138	0.225	0.197	0.280	0.368	0.546
				$\phi = 0.20$				
Beta	1.000	1.004	1.009	1.018	0.915	0.916	0.916	0.917
St dev	0.000	0.103	0.161	0.265	0.201	0.304	0.409	0.616
	*C. Joint Jumps Only****				*D. Disjoint Jumps Only*****			
				$\phi = 0.00$				
Beta	1.000	1.000	1.000	1.000	0.722	0.721	0.720	0.717
St dev	0.000	0.000	0.000	0.000	0.424	0.564	0.719	1.029
				$\phi = 0.05$				
Beta	1.000	1.000	1.000	1.000	0.731	0.730	0.730	0.727
St dev	0.000	0.084	0.131	0.215	0.424	0.574	0.735	1.061
				$\phi = 0.10$				
Beta	1.000	1.000	1.000	1.001	0.740	0.740	0.740	0.740
St dev	0.000	0.125	0.195	0.322	0.424	0.586	0.758	1.105
				$\phi = 0.15$				
Beta	1.000	1.000	1.001	1.002	0.748	0.749	0.750	0.753
St dev	0.000	0.163	0.253	0.409	0.423	0.595	0.774	1.144
				$\phi = 0.20$				
Beta	1.000	1.000	1.001	1.002	0.757	0.758	0.760	0.764
St dev	0.000	0.197	0.306	0.499	0.423	0.606	0.795	1.178

Note: ϕ is the asynchronicity parameter of futures. *Jump size parameters of equation (17) are $\alpha_1 = \alpha_2 = \alpha_3 = 0$. **Jump size parameters of equation (17) are$\alpha_1 = \alpha_2 = \alpha_3 = 0.6$. ***Jump size parameters of equation (17) are $\alpha_3 = 0.6$ and $\alpha_1 = \alpha_2 = 0$. ****Jump size parameters of equation (17) are $\alpha_3 = 0$ and $\alpha_1 = \alpha_2 = 0.6$. 5000 joint jumps, 1000 disjoint jumps (500 disjoint jumps in futures, 500 disjoint jumps in spot).

Table 8: Mean and Standard Deviation of Beta Across Sampling Frequencies, 1000 Replications Over 1000 Days, DGP With or Without Jumps, Ratio of Joint to Disjoint Jumps is 10:1

Sampling Frequency	**5 min**	**30 min**	**60 min**	**120 min**	**5 min**	**30 min**	**60 min**	**120 min**
	*A. Without Jumps**				*B. With Jumps***			
	$\phi = 0.00$							
Beta	1.000	1.000	1.000	1.000	0.953	0.953	0.954	0.953
St dev	0.000	0.000	0.000	0.000	0.148	0.183	0.224	0.317
	$\phi = 0.05$							
Beta	1.000	1.001	1.002	1.005	0.953	0.953	0.953	0.953
St dev	0.000	0.050	0.077	0.125	0.152	0.217	0.289	0.435
	$\phi = 0.10$							
Beta	1.000	1.002	1.005	1.010	0.953	0.953	0.954	0.954
St dev	0.000	0.071	0.110	0.181	0.155	0.257	0.352	0.550
	$\phi = 0.15$							
Beta	1.000	1.003	1.007	1.014	0.954	0.954	0.954	0.955
St dev	0.000	0.088	0.138	0.225	0.158	0.295	0.421	0.670
	$\phi = 0.20$							
Beta	1.000	1.004	1.009	1.018	0.954	0.954	0.954	0.955
St dev	0.000	0.103	0.161	0.265	0.162	0.342	0.494	0.780
	*C. Joint Jumps Only****				*D. Disjoint Jumps Only*****			
	$\phi = 0.00$							
Beta	1.000	1.000	1.000	1.000	0.818	0.817	0.816	0.815
St dev	0.000	0.000	0.000	0.000	0.435	0.706	0.971	1.485
	$\phi = 0.05$							
Beta	1.000	1.000	1.000	1.000	0.825	0.824	0.825	0.829
St dev	0.000	0.113	0.178	0.297	0.435	0.719	0.993	1.524
	$\phi = 0.10$							
Beta	1.000	1.000	1.000	1.001	0.832	0.832	0.834	0.838
St dev	0.000	0.170	0.265	0.441	0.435	0.738	1.025	1.586
	$\phi = 0.15$							
Beta	1.000	1.001	1.001	1.002	0.838	0.840	0.843	0.848
St dev	0.000	0.224	0.349	0.578	0.437	0.758	1.054	1.629
	$\phi = 0.20$							
Beta	1.000	1.000	1.001	1.002	0.846	0.848	0.851	0.857
St dev	0.000	0.281	0.437	0.712	0.436	0.774	1.084	1.685

Note: ϕ is the asynchronicity parameter of futures. *Jump size parameters of equation (17) are $\alpha_1 = \alpha_2 = \alpha_3 = 0$. **Jump size parameters of equation (17) are$\alpha_1 = \alpha_2 = \alpha_3 = 0.6$. ***Jump size parameters of equation (17) are $\alpha_3 = 0.6$ and $\alpha_1 = \alpha_2 = 0$. ****Jump size parameters of equation (17) are $\alpha_3 = 0$ and $\alpha_1 = \alpha_2 = 0.6$. 5000 joint jumps, 500 disjoint jumps (250 disjoint jumps in futures, 250 disjoint jumps in spot).

Table 9: Mean and Standard Deviation of Beta Across Sampling Frequencies, 1000 Replications Over 1000 Days, DGP With or Without Jumps, Correlation Between Assets is 0.5

Sampling Frequency	**5 min**	**30 min**	**60 min**	**120 min**	**5 min**	**30 min**	**60 min**	**120 min**
	*A. Without Jumps**				*B. With Jumps***			
				$\phi = 0.00$				
Beta	0.498	0.510	0.524	0.549	0.900	0.900	0.900	0.901
St dev	0.084	0.206	0.304	0.487	0.201	0.249	0.304	0.425
				$\phi = 0.05$				
Beta	0.498	0.510	0.525	0.552	0.899	0.898	0.899	0.899
St dev	0.086	0.212	0.315	0.505	0.206	0.277	0.355	0.527
				$\phi = 0.10$				
Beta	0.499	0.511	0.526	0.554	0.898	0.898	0.898	0.898
St dev	0.088	0.219	0.326	0.524	0.211	0.308	0.412	0.618
				$\phi = 0.15$				
Beta	0.499	0.512	0.527	0.556	0.896	0.897	0.897	0.898
St dev	0.090	0.227	0.337	0.544	0.217	0.345	0.474	0.722
				$\phi = 0.20$				
Beta	0.498	0.512	0.528	0.559	0.894	0.895	0.896	0.897
St dev	0.093	0.234	0.349	0.564	0.223	0.385	0.538	0.835
	*C. Joint Jumps Only****				*D. Disjoint Jumps Only*****			
				$\phi = 0.00$				
Beta	0.986	0.986	0.986	0.987	0.334	0.343	0.352	0.368
St dev	0.056	0.065	0.077	0.103	0.384	0.827	1.200	1.897
				$\phi = 0.05$				
Beta	0.984	0.984	0.985	0.985	0.339	0.348	0.358	0.376
St dev	0.061	0.132	0.195	0.311	0.389	0.850	1.235	1.950
				$\phi = 0.10$				
Beta	0.982	0.983	0.983	0.984	0.345	0.352	0.362	0.379
St dev	0.068	0.188	0.287	0.453	0.401	0.884	1.284	2.022
				$\phi = 0.15$				
Beta	0.980	0.980	0.981	0.982	0.351	0.361	0.372	0.392
St dev	0.074	0.242	0.368	0.592	0.407	0.908	1.324	2.085
				$\phi = 0.20$				
Beta	0.978	0.978	0.979	0.981	0.356	0.365	0.377	0.396
St dev	0.080	0.295	0.447	0.719	0.417	0.933	1.367	2.162

Note: ϕ is the asynchronicity parameter of futures. *Jump size parameters of equation (17) are $\alpha_1 = \alpha_2 = \alpha_3 = 0$. **Jump size parameters of equation (17) are$\alpha_1 = \alpha_2 = \alpha_3 = 0.6$. ***Jump size parameters of equation (17) are $\alpha_3 = 0.6$ and $\alpha_1 = \alpha_2 = 0$. ****Jump size parameters of equation (17) are $\alpha_3 = 0$ and $\alpha_1 = \alpha_2 = 0.6$. 5000 joint jumps, 1000 disjoint jumps (500 disjoint jumps in futures, 500 disjoint jumps in spot).