Recent Developments in Dynamic Portfolio Choice

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QUT
28 November 2012
Recent Developments

- Multiple risky assets with predictable returns and trading costs: Lynch and Tan (2010) JFQA.

- State-dependent asset returns and labor income growth: Lynch and Tan (2011) JFE.

- State-dependent asset returns, labor income growth, and trading costs:
  - Liquidity premia calculation.
  - Lynch and Tan (2011) JF.
Multiple Risky Assets, Transaction Costs and Return Predictability: Implications for Portfolio Choice

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Introduction

- Our analysis considers an economic environment with:
  - transactions costs.
  - return predictability.
  - more than one risky asset.
- Characterize decision-making by a dynamic multi-period CRRA investor.
- Use numerical methods.
- Focus on the two risky asset case.
- Use analysis to examine economic questions:
  - what is the utility cost of redemption fees?
  - what is the utility benefit of being able to borrow?
  - what is the utility benefit of being able to short and how much does this benefit get reduced if the stocks shorted are ”special”?
  - a horse race between mutual funds (no shorting but low transactions costs), ETFs (shorting but return reduced by expenses), and individual stocks (shorting but high transactions costs).
  - what is the utility cost of using only one risky portfolio instead of two?
  - what is the utility cost of financial service charges (transaction costs, expenses and redemption fees)?
Outline

- Relation to Literature
- Motivation
- Limitations
- Decision-making Model
- Portfolio Allocation Rules
- Calibration of Model
- Economic Questions
Relation to Literature

◦ Portfolio Choice with Transactions Costs

  ◦ Many Risky Assets: all papers consider a constant opportunity set
    ◦ Liu (2004): CARA agent, analytical for uncorrelated returns and numerical for 10% correlation, assumes form of the solution
    ◦ Aakian, Menaldi and Sulem (1996): CRRA agent, uncorrelated returns, existence and uniqueness and some numerical results.

  ◦ Single Risky Asset: better understood
    ◦ Constantinides (1986): optimal policy characterized by a no-trade region with return to the closer boundary when rebalancing.

◦ Portfolio Choice with Margin Requirements

  ◦ Cuoco and Liu (2000): use martingale and duality techniques to establish existence of optimal consumption and investment policies.
Motivation

- Investors do face transactions costs.
- Investors do trade many assets.
- Returns do seem predictable.

Limitations

- Numerical results only.
Decision-making Model

○ Wealth Evolution:

\[ W_{t+1} = (W_t - C_t) \left( \sum_{i=1}^{N} \alpha_{i,t} R_{t+1}^i + \alpha_{f,t} R^f \right) = W_t(1 - \kappa_t)P_{t+1}^W, \tag{1} \]

and

\[ \alpha_{f,t} = 1 - \sum_{i=1}^{N} \alpha_{i,t} - f_t, \tag{2} \]

○ \( f \) is the transactions cost per dollar of portfolio value: paid out of the riskless asset.

○ \( \alpha \) is a vector of portfolio allocations: unlimited shorting and borrowing.

○ Cost Function:

\[ f = \phi_p' |\alpha - \hat{\alpha}| \tag{3} \]

○ \( \phi_p \) is a vector of proportional transactions costs parameters.

○ Can easily incorporate costs that are a fixed fraction of portfolio value.

○ Inherited Allocation Evolution:

\[ \hat{\alpha}_{t+1}^i \equiv \frac{\alpha_{t}^i R_{t+1}^i}{P_{t+1}^W} \tag{4} \]

○ where \( \hat{\alpha}_{t}^i \) is the allocation to the \( i \)th risky asset inherited from time-(\( t - 1 \)).

○ Note: \( C_t \) is obtained by liquidating costlessly the \( i \)th risky asset and the riskless asset in the proportions \( \hat{\alpha}_{t}^i \) and \( (1 - \hat{\alpha}_{t}^i)N \).

○ Returns

\[ r_{t+1} = a_r + b_r d_t + e_{t+1} \tag{5} \]

\[ d_{t+1} = a_d + b_d d_t + v_{t+1} \]
Decision-making Model (cont)

- Preferences:
  \[
  E \left[ \sum_{t=1}^{T} \delta^t \frac{c_{1}^{1-\gamma}D_1}{1 - \gamma} | \hat{D}_1, \hat{\alpha}_1 \right]
  \] (6)

- Time separable and constant relative risk aversion.

- Bellman Equation:
  \[
  \frac{a(D_t, \hat{\alpha}_t, t)W_t^{1-\gamma}}{1 - \gamma} = \max_{\kappa_t, \alpha_t} \frac{\kappa_t^{1-\gamma}W_t^{1-\gamma}}{1 - \gamma} + \frac{(1 - \kappa_t)^{1-\gamma}W_t^{1-\gamma}}{1 - \gamma} E \left[ a(D_{t+1}, \hat{\alpha}_{t+1}, t + 1)R_{w,t+1}^{1-\gamma} | D_t, \hat{\alpha}_t \right],
  \] for \( t = 1, \ldots, T - 1 \), (7)

- Value function homogeneous of degree \((1 - \gamma)\) in wealth.

- Inherited allocations of assets with non-zero transaction costs are state variables.

- Solution Technique:
  - Discretize state and action spaces: use the same 0%, 1%, 2%, ... , 99%, 100% grid for each.
  - Interpolate value function over the state space using its natural triangular tesellation: collapses to linear interpolation when only one asset’s allocation is a state variable.
  - Develop an algorithm that exploits global concavity of the value function.
○ Extensions:

  ○ Impose margin requirements on shorting and borrowing.
  ○ Drive a wedge between the lending and borrowing rates.
  ○ Allow the rebate rate for shorting to be less than $R^f$.
  ○ Incorporate a redemption fee for selling an asset within $n$ months of purchase.
Labor Income Dynamics at Business-cycle Frequencies: Implications for Portfolio Choice

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28 November 2012

Recent Developments in Dynamic Portfolio Choice

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Motivation

- Impact of Hedging Demands Induced by Business-cycle Variation in Labor Income Growth: Not Well Understood

- Empirical Literature on Portfolio Holdings of U.S. Consumers vs Theoretical Portfolio-choice Literature with Labor Income: Completely Opposite Conclusions

  - Empirical:
    - Young: Rich hold a larger fraction in stock than Poor
    - Young hold a smaller fraction in stock than Old
    - Substantial non-participation, especially for the Poor

  - Theoretical:
    - Young: Rich hold a smaller fraction in stock than Poor
    - Young hold a larger fraction in stock than Old
    - Everyone participates
Stylized Facts About Individual Labor Income From US Data

- First two moments of individual income growth move with the business cycle
  - Intuition suggests **pro-cyclicality in expected income growth**: Point estimates are significant and confirm this (SD\text{Mean})
  - Storesletten, Telmer and Yaron (STY, 04): strong **counter-cyclicality in volatility of labor income growth** (SD\text{Vol})

- **Contemporaneous correlation** between labor income growth and stock return: small positive (Davis and Willen, 02) (Cor\text{WR})

**Current Theoretical Literature Fails to Capture these Stylized Facts**

- Assumes **shocks to labor income growth and stock return are i.i.d.**
- Further, correlation between two is either artificially high and positive or zero

**This Paper Captures these Stylized Facts**

- Assumes a **joint process for labor income and stock return consistent with stylized facts** and uses simple VAR dynamics to switch each of the three ON or OFF
Main Results

- Allowing for business-cycle variation in individual income growth puts theory in line with the stylized facts
  - Among Young: Rich hold a larger fraction in stock than Poor
  - Young hold a smaller fraction in stock than Old
  - Substantial non-participation, especially for the Poor and Young

Main Contributions

- Recognize the importance of negative hedging demands induced by business cycle variation in labor income growth
- Quantify the magnitude of these negative hedging demands
- Show these hedging demands bring the model closer to data
- Show contemporaneous income growth return correlation has negligible effect when correlation is calibrated to data
Illustration Using 20-year Case: Magnitudes of Hedging Demands are Large

- Agent:
  - Has access to a market portfolio of stocks and a riskless T-bill
  - Receives labor income calibrated to a typical wage earner in the “Retail Trade” industry

- Stock return predictability: calibrated to data

- 20 year horizon with monthly rebalancing

- Power utility with relative risk aversion 6
Quantitative Results for Young: Rich vs Poor

○ Consider Young agents with financial wealth to monthly labor income ratios of 1, 10, 1000

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<thead>
<tr>
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<th>Poor</th>
<th>Rich</th>
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○ **Current Literature** (iid income growth uncorrelated with stock return): Higher fractions in stock for Poor than Rich

  ○ All 3 channels switched OFF 97.5% 95.5% 56.9%

○ **Business-cycle variation** in the 1st 2 moments of income growth as in data: Lower fractions in stock for Poor than Rich

  ○ Add SDMean and SDVol 23.2% 25.8% 51.5%

○ **Positive correlation** of income growth and stock return as in data: Almost no effect on allocations

  ○ Add SDMean, SDVol and CorWR 22.0% 25.2% 51.4%

○ Both the first and second moment predictability are contributing to reductions

  ○ Add SDMean alone 86.0% 82.6% 52.5%

  ○ Add SDVol alone 27.2% 27.3% 55.6%
Quantitative Results for Young vs Old

- Consider simulation in which Young agent’s wealth-income ratio is 30

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<tr>
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<th>Young</th>
<th>Old</th>
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<tr>
<td>All 3 channels switched OFF</td>
<td>91%</td>
<td>72%</td>
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- **Current literature**: Young have larger allocation to stock than Old

- **Business-cycle variation** in the 1st 2 moments of income growth as in data: Young have smaller allocation to stock than Old

  - Add $SDMean$ and $SDVol$ | 38%   | 74% |
Quantitative Results for Non-participation

- Consider Young agents with financial wealth to monthly labor income ratios of 1, 10, 1000
- Examine proportion of time a Young agent does not participate in the stock market

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- **Current Literature**: A Young agent participates almost all the time
  - All 3 channels switched OFF: 2% 0% 0%

- **Business-cycle variation** in the 1st 2 moments of income growth calibrated to data: Substantial non-participation especially for the Poor
  - Add SDMean and SDVol: 73% 67% 0%
Intuition for Reduction in Stock Holding by *Young and Poor*

- **Theoretically**, Merton (1973) for risk aversion greater than 1:
  - Positive correlation between return and future investment opportunities leads to reductions in stock holdings by *Young*

- **Empirically**:
  - Realized stock return is low when the probability of a recession increases
  - But in recessions expected income growth is low and volatility of income growth is high
  - So a *low* stock return this period means increased probability of *low* expected income growth and *high* volatility of income growth in the next period and future periods
  - So stock returns and future “labor income” opportunities are positively correlated

- **Therefore** business-cycle variation in first two moments of income growth:
  - Causes reductions in stock holdings by the *Young*
  - Reductions are more pronounced for the *Poor*, for whom future labor income is more important
Robustness Checks

○ Calibration of the business-cycle variation in permanent labor income volatility:

    ○ $SDVol$ channel’s effect on allocations robust to using the NBER variable directly to calibrate the expansion-recession state

○ Temporary shocks to labor income: Negligible effect on allocations

○ Transitory or Persistent unemployment state: Persistent also reduces stock allocations

○ Hump shaped working life profile for earnings and retirement
Other Related Literature

- **Empirical Literature**: Bertaut (94), Blume and Zeldes (94), Friend and Blume (75), Heaton and Lucas (00), Poterba (93), Vissing-Jorgensen (02), Ameriks and Zeldes (01), Calvet, Campbell and Sodini (06)

- **Theoretical Literature**: Viceira (01), Cocco, Gomes and Maenhaut (05), Davis and Willen (00), Heaton and Lucas (97), Heaton and Lucas (2000), Michaeliedes (03), Benzoni, Collin-Dufresne and Goldstein (06)

- **General Equilibrium** models with countercyclical idiosyncratic income risk
  - Constantinides Duffie (96), Heaton Lucas (96)

- **Other explanations** for low stock holdings: none a complete explanation by itself
  - **Participation costs**: Vissing-Jorgensen (02)
  - **Additive internal habit**: Polkovnichenko (03)
  - **Housing**: Cocco (04), Yao and Zhang (02,04)
Decision-making Model with Labor Income: Exogeneous Processes

**Labor Income**

- As in Carroll (96) and (97)

\[ y_{t+1} = y_{t+1}^P + \epsilon_{t+1} \]  
\[ g_{t+1} \equiv \Delta y_{t+1}^P = \bar{g} + b_g d_t + u_{t+1} \]

- \( y \) is log labor income, \( y^P \) is log permanent income, \( \epsilon \) is log temporary labor income
- \( d \) is the mean reverting predictor to proxy for the business cycle taken to be the dividend yield
- \( \epsilon_t \) and \( u_{t+1} \) are uncorrelated i.i.d. processes
- \( \sigma_{u_{t+1}} \) can be a function of \( d_t \) to allow for heteroskedasticity of permanent income growth

**Returns and Dividend Yield**

- As in Campbell and Viceira (99) and Lynch (01)

\[ r_{t+1} = a_r + b_r d_t + e_{t+1} \]  
\[ d_{t+1} = a_d + b_d d_t + w_{t+1} \]
Decision-making Model with Labor Income: Specifications Considered

○ **20-year Specification:**
  ○ 20 year working life then the terminal date

○ **Life-cycle Specification:**
  ○ 43 year working life followed by 35 years of retirement
  ○ Mortality rates calibrated to US data
  ○ Social Security Benefits: Annuity income throughout retirement
  ○ Earnings Profile: Hump-shaped
Decision-making Model with Labor Income: Recursive Formulation, 20-year Specification

◦ Financial Wealth Evolution:

\[ W_{t+1} = (W_t + Y_t - C_t) \left[ \alpha_t (R_{t+1} - R^f) + R^f \right] \]

for \( t = 1, \ldots, T - 1 \) \hspace{1cm} (5)

◦ Terminal Condition: \( C_T = W_T + Y_T \)

◦ Objective Function:

\[
\max_{\{C_t, \alpha_t \in F_t\}_{t=1}^{T-1}} E \left[ \sum_{t=1}^{T} \delta^t \frac{C_t^{1-\gamma}}{1-\gamma} \bigg| F_1 \right].
\] \hspace{1cm} (6)

◦ No short selling of stock or borrowing

◦ Value function:

\[
V_t \equiv \frac{a(W_t/Y_t^P, d_t, t)(Y_t^P)^{1-\gamma}}{1-\gamma}
\] \hspace{1cm} (7)

○ \( W_t/Y_t^P \) and \( d_t \) are state variables

○ Recursion is solved by iterating backward
Numerical Solution

○ Difficult features of the problem:

○ Wealth to period labor income is a continuous state variable with range $[0, \infty]$

○ Value function has high curvature for low wealth/income values

○ Literature:

○ Heaton and Lucas (97): report that results can be sensitive to the approximations used

○ Our paper:

○ Suggests and implements formal procedures to ensure results are accurate despite the approximations used
Transaction Costs, Return Predictability and Wealth shocks: Implications for Liquidity Premia

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28 November, 2012
Introduction and Motivation

- Impact of transactions costs on mean returns.
  - Theory: Constantinides (1986).
    - Liquidity premium:
      - Is the difference in the expected returns on two otherwise identical assets,
      - Only one of which is subject to transactions costs,
      - Such that an investor is indifferent as to which asset she has access to.
  - Standard problem with i.i.d. equity returns and proportional transaction costs: Per-annum liquidity premium is an order of magnitude smaller than the transaction costs rate itself.
  - Empirical:
    - Large differences in mean per-annum risk-adjusted returns of liquidity measure-sorted portfolios: Same order of magnitude as the differential in the transaction cost rate.
      - Pástor and Stambaugh (2002): 7.5% p.a. ab. ret. spread for covariance with illiquidity deciles.
    - Need to connect and reconcile these seemingly contradictory literatures.
    - One direction is to examine more realistic decision-making models.
Our Work

○ Complicate the standard i.i.d. problem by introducing:
  ○ Return predictability: calibrated to U.S. data.
  ○ State dependent wealth shocks:
    ○ Labor income: Calibrated to U.S. data.
    ○ Stationary multiplicative.
    ○ State dependent transactions cost rates: Means calibrated to U.S. data.
  ○ Keep the problem multi-period: Which is important (Constantinides, 1986).
  ○ Allow access to a second high liquidity asset: Important since this access reduces liquidity premia on the low liquidity asset.
  ○ Compare the magnitudes of the implied liquidity premia to that for the standard i.i.d. problem.
Calibration

○ Cost rate calibration: Based on rates estimated for the 5 smallest and the 5 largest size deciles (Lesmond, Trzcinka and Ogden, 1999)
  ○ Low liquidity portfolio: 3%.
  ○ High liquidity portfolio: 1%.

○ Return calibration: Based on portfolios of NYSE and AMEX stocks formed on basis of Amihud’s (2002) illiquidity measure.
  ○ i.i.d..
  ○ Predictable using dividend yield.

○ Labor income calibration: Based on PSID data for professionals and managers not self-employed under the age of 45.
  ○ Growth in permanent component allowed to be pro-cyclical as in data.
  ○ Predictable using dividend yield.
Main Result

○ Add real world complications to the canonical problem; Then per-annum liquidity premia:

  ○ No longer an order of magnitude smaller than the cost rate.
  ○ Now can be the same order of magnitude as the cost rate.

○ Calibration results:

  ○ Agent has access to a riskless asset, the value weighted portfolio of the 13 least liquid (Low Liquidity Portfolio) and the 12 most liquid (High Liquidity Portfolio) of 25 liquidity sorted portfolios.
  ○ Transactions cost rate on the low liquidity portfolio is a constant 2%, the cost spread in the data between the low and high liquidity portfolios.
  ○ Consider a financial wealth to monthly permanent labor income ratio of 10 in labor income cases.
  ○ Liquidity premia:

    ○ 0.08 % p.a. with i.i.d. returns and no-labor income growth.
    ○ 0.94 % p.a. with i.i.d. returns and i.i.d. labor income growth.
    ○ 1.12 % p.a. with predictable returns and procyclical labor income growth (Base Case).

○ Premia numbers are close to the Fama-French abnormal return differential between the low and high liquidity portfolios of 1.88 % p.a..
Comparison with the Standard I.i.d. Problem

○ Constantinides (86)
  ○ For the same agent facing no transaction costs: Has the same optimal allocation every period.
  ○ Shocks to portfolio compositions due to: Realized returns on component assets.
  ○ Transactions cost rate: Constant.

○ Our Set-up
  ○ For the same agent facing no transaction costs:
    ○ Return predictability means the optimal allocation changes with the state each period.
    ○ Hedging demands mean that the optimal allocation changes with age holding the state fixed.
  ○ Shocks to portfolio compositions due to:
    ○ Realized returns on component assets.
    ○ Realized cash wealth shocks.
  ○ Transactions cost rate:
    ○ Unconditional mean is calibrated to data as in Constantinides.
    ○ But the rate can be higher in states in which transaction costs hurt more.
Interpreting the Liquidity Premium for a Given Agent

○ Three possibilities for a given agent:

  ○ Marginal agent (holds the asset and sets the price): Agent liquidity premium is the equilibrium liquidity premium.

  ○ Inframarginal agent (holds the asset but is not marginal): Agent liquidity premium is a lower bound on the equilibrium liquidity premium.

  ○ Agent does not hold the asset: Agent liquidity premium says nothing about the equilibrium liquidity premium.

○ Want to choose an agent whose likely to be holding the asset in question.
Interpreting the Base Case Liquidity Premia Results

- Base-case agent allocates more to the low than the high liquidity portfolio but in the U.S., market cap is lower for the low than the high liquidity portfolio.
  - So base-case agent appears to be an inframarginal agent: Her liquidity premium likely represents a lower bound on the equilibrium liquidity premium.
- Heterogeneity across agents must be important for all assets to be held, consistent with market clearing.
- Delegated portfolio managers can be shown to tilt their portfolios towards the benchmark assets used to determine fund fees (Cuoco and Kaniel, 2006).
  - Benchmark assets are typically liquid.
  - So funds managed on behalf of others hold large fractions of their portfolios in liquid assets.
  - Delegated portfolio managers also help markets to clear with all assets being held.
- Idiosyncratic variation in permanent income constitutes almost all permanent income variation.
  - Trades motivated by changes in human capital value are largely unsynchronized.
  - Net trades can be zero, consistent with market clearing.
Limitations

- Partial equilibrium.

- Says nothing about how transactions costs affect market clearing prices via their effect on risk-sharing.

- First pass towards an understanding of how equilibrium prices and returns might be affected.
Relation to Literature

◦ Seminal work by Constantinides (1986): Calibration result.

◦ Other work:

  ○ Heaton and Lucas (1996): Two classes of agent face idiosyncratic labor income risk and transaction costs to trade the risky and riskless assets.

  ○ Vayanos (1998): Agents trade for life-cycle purposes and all trading is predetermined.

  ○ Aiyagari and Gertler (1991): Economy has idiosyncratic shocks but no aggregate uncertainty.

  ○ Huang (2002): i.i.d. liquidity shocks lead to higher premia for riskless assets.

  ○ Acharya and Pedersen (2002): All pairwise covariances of asset return and liquidity and market return and liquidity can affect expected asset return.

Decision-making Model with Labor Income

○ Preferences: Time separable and constant relative risk aversion.

\[ E_t \left[ \sum_{t=1}^{T} \delta^t \frac{c_t^{1-\gamma}}{1 - \gamma} \right] \]  \hspace{1cm} (1)

○ Labor income process (follow Carroll 1996 and 1997):

\[ y_t = y_t^P + \epsilon_t, \]  \hspace{1cm} (2)

\[ g_t \equiv y_t^P - y_{t-1}^P = \bar{g} + b_y d_t + u_{t+1}. \]  \hspace{1cm} (3)

○ \( y_t \) is log labor income, \( y_t^P \) is log permanent labor income and \( d_t \equiv \ln(D_t) \).

○ \( \epsilon_{t+1} \) is log temporary labor income, and \( \epsilon_t \) and \( u_{t+1} \) are uncorrelated i.i.d. processes.

○ Wealth evolution:

\[ W_{t+1} = (W_t + Y_t - c_t)(1 - f_t) \left[ \alpha_t' (R_{t+1} - R_t^f i_N) + R_t^f \right] \]

\[ \text{for } t = 1, \ldots, T - 1, \]  \hspace{1cm} (4)

○ \( Y_t \) is labor income received at time \( t \).

○ dollar transaction costs are \((W_t + Y_t - c_t)f_t\).
Decision-making Model with Labor Income (cont)

◦ Cost Function: \( f_t = \Phi_t' | \alpha_t - \hat{\alpha}_t W_t |. \)

  ○ Labor income is assumed to be received as cash.

  ○ Allowed to be stochastic:

\[
\log(1 + \Phi_t) = a_\phi + b_\phi d_t + \omega_{\phi,t+1}.
\]

(5)

◦ Transaction Costs: Paid at time \( t \) by costlessly liquidating \( i \)th risky and riskless assets in the proportions \( \alpha_i^t \) and \( (1 - \alpha_i^t)N \).

◦ Consumption at \( t \): Liquidate costlessly \( i \)th risky and riskless assets in the proportions \( \hat{\alpha}_i^t \) and \( (1 - \hat{\alpha}_i^t)N \).

◦ No short selling.
Decision-making Model with Labor Income (cont)

◦ Evolution of Portfolio Proportions:

\[ \hat{\alpha}_{t+1}^i \equiv \frac{\alpha_t^i R_{t+1}^i}{\alpha_t'(R_{t+1} - R_{i N}^f)} + R_{t}^f \]  

\hspace{10cm} (6)

◦ Value function: \[ \frac{a(\Gamma_t, D_t, \hat{\alpha}_t, t)(Y_{t-1}^P)^{1-\gamma}}{1-\gamma} \].

  ○ Homogeneous in \( Y_{t-1}^P \).

  ○ State variables: \( D_t, \hat{\alpha}_t \) and \( \Gamma_t \equiv W_t / Y_{t-1}^P \).

◦ Bellman Equation:

\[ a(\Gamma_t, D_t, \hat{\alpha}_t, t)(Y_{t-1}^P)^{1-\gamma} \]

\[ \hspace{1cm} 1 - \gamma \]

\[ = E \left[ \max_{\hat{\kappa}(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t), a(\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t, t)} \left\{ \hat{\kappa}_t^{1-\gamma}(Y_{t-1}^P)^{1-\gamma} \right\} \right] \\
\hspace{1cm} + \delta \frac{(Y_{t-1}^P)^{1-\gamma}}{1-\gamma} E \left[ a(\Gamma_{t+1}, D_{t+1}, \hat{\alpha}_{t+1}, t+1)(\exp\{g_t\})^{1-\gamma}|\Gamma_t, D_t, \hat{\alpha}_t, g_t, \epsilon_t, \Phi_t \} \right| \Gamma_t, D_t, \hat{\alpha}_t \right] , \]

\hspace{1cm} for \( t = 1, \ldots, T - 1 \),  

\hspace{10cm} (7)
Remark on Numerical Implementation of Labor Income Problem

○ Novel features:

○ Endogenous discrete state representation of the value function (as suggested by Gourinchas and Parker (2002)).

○ No extrapolation: Exploit convergence to an easily solved problem with no labor income as \( W/Y \) goes to infinity.
Future Directions: Theory

◦ Cross-sectional variation in labor income processes: Implications for the cross section of portfolio choices.

◦ Parameter uncertainty and learning:
  ◦ Dynamic portfolio choice without learning in a discrete-time setting: Barberis (2000) JF.
  ◦ Learning in a continuous-time setting: Xia (2001) JF.

◦ How housing and labor income together affect dynamic portfolio choices:
  ◦ Household choice between a fixed-rate (FRM) and an adjustable-rate (ARM) mortgage: Cocco and Campbell (2003) QJE.
  ◦ Dynamic portfolio choice with housing, labor income and i.i.d. stock returns: Cocco (2005) RFS.
Future Directions: Data

- Comprehensive Swedish data set:
  - Efficiency of household portfolios in a static setting: Calvet, Campbell, and Sodini (2007) JPE.
  - Time-series dynamics of individual portfolios: Calvet, Campbell, and Sodini (2009) QJE.