Extending an SVAR Model of the Australian Economy

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Abstract

Dungey and Pagan (2000) present an SVAR model of the Australian economy which models macro-economic outcomes as transitory deviations from a deterministic trend. In this paper we extend that model in two directions. Firstly, we relate it to an emerging literature on DSGE modelling of small open economies. Secondly, we allow for both transitory and permanent components in the series and show how this modification has an impact upon the design of macroeconomic models.

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1 Introduction

The interaction of theory and evidence is a crucial part of applied macroeconomic research. Empirical models should reflect theoretical models in their design while the design itself should also react to what is in the data. Increasingly model design has become focussed upon the class of New Keynesian (NK) and Dynamic Stochastic General Equilibrium (DSGE) models. Although these are also often used directly in empirical work, it is probably the case that Structural VARs are still regarded as the best way to discover what dynamic relations exist between multivariate series. Nevertheless, it is desirable if SVARs broadly incorporate the structures that come from the theoretical models.

In the 1990s we built a small Australian Structural VAR (SVAR) macroeconometric model - Dungey and Pagan (2000)- which will be referred to as the SVAR2000 model. The emphasis in SVAR2000 was upon data coherency, with the consequence that theoretical ideas appeared in it in only a loose way. For this reason we wish to now examine the extent to which it is compatible with NK and DSGE models. In doing so, we also want to ask how the data we work with should determine the design of our model. We argue that there are unit roots in the data and co-integration between some variables. Investigation of this feature shows that the way many NK and DSGE models are implemented on such data fails to adequately reflect this feature.

The basic idea behind our original paper was that data on a set of \( n \) variables that are to be modelled could be well represented by a \( VAR(p) \) process. These variables were of two types - those that could be thought of as determined in the "rest of the world" and those that were best regarded as being domestically determined. The former variables were US GDP, the real Dow-Jones Index, the terms of trade, exports and the real US interest rate, and these were regarded as being strongly exogenous to the Australian economy. The latter were the All-Ordinaries Index, Gross National Expenditure, GDP, the underlying inflation rate, the policy interest rate instrument and
the real exchange rate. We then loosely used theoretical ideas to make this a Structural VAR (SVAR) model. As we will see in the next section, theoretical models of the NK variety imply that, if the data are $I(0)$, they should follow a structural VARX (SVARX) model, where the X identifier indicates there are exogenous variables that are conditioned upon. Given the strong exogeneity assumptions in force, SVAR2000 can also be thought of as an SVARX model. Our first step in this paper then is to specifically re-formulate SVAR2000 to produce SVARX2000 so as to make a proper comparison to the newer model.

Now it seems as if there are likely to be unit roots and co-integration among some of the variables in our data set. Indeed, DSGE models are often designed to reflect this fact. To obtain an SVAR representation it is conventional to transform the $I(1)$ variables in the models into "gap" form by scaling any $I(1)$ processes with their permanent components. These gap variables are then $I(0)$ processes and follow an SVAR. But when implemented on data it is often the case that NK and DSGE models are simply estimated using data that has been transformed to $I(0)$ form through some filtering operation, such as the Hodrick-Prescott filter, which generally does not reproduce the model-consistent estimate of the permanent component. Both of these strategies are in contrast to what happens with models that are loosely based on economic reasoning. In these situations the SVAR is replaced by a Structural Vector Error Correction Model (SVECM) and the model design derives from this structure. Section 3 therefore looks at the relation between models expressed as "gaps" and the SVECM construct. We find that, when there is co-integration, we generally cannot reduce the SVECM to an SVAR in "gaps", unless one is careful about what is included in the SVAR taken to represent NK and DSGE models.

The interaction between empirical and theoretical models is not just about model design however. There are often elements in models such as SVAR2000 which seem important to understanding the economy under investigation but which are either absent or hard to integrate into small theoret-
ical models. This is true of the SVAR2000 model, and also of our extended model, termed H2007. The latter has a theoretical orientation but which reacts to the nature of the data, specifically the fact that there seems to be co-integration among the variables. A conflict between models implied by empirical and theoretical perspectives might also arise from a number of other sources. First, more variables may enter into empirical models, since no simple account of the determination of the extra variables is provided in any existing theoretical model, and so they are often excluded from the latter. Second, the order of the SVARXs can differ, as the data might suggest more complicated dynamics than is available from a theoretical model. Thirdly, the assumption of rational expectations in theoretical models suggests that expectations can be constructed by applying a specific set of weights to the information they are based on, whereas empirical models are more iconoclastic and allow for unrestricted weights. Section 4 first looks at some of these issues in the context of SVARX2000 before discussing the number and nature of co-integrating relations in the data, as the latter determines the SVECMA that underlies H2007. We try to motivate these co-integrating relations from the perspective of a DSGE model, although this is not entirely easy. Finally, Section 5 estimates H2007, looks at the impulse responses that come from it, and compares them to those from SVARX2000. It transpires that there are significant differences between the two, which must derive from the influence of the assumptions made about the presence of permanent shocks in one model but not the other.

2 Model Design with I(0) Variables

A model that has inspired many small macro models is the New Keynesian-Model (NK) for an open economy. Examples are Svensson (2000), Justiniano and Preston (2006), Lubik and Schorfheide (2005), Gali and Monacelli (2005), Giordani (2004) and Nimark(2007). The following equations summarize its
essence, although some of the models mentioned above are larger than this stylized version e.g. Nimark (2007). In order, they include an IS curve describing absorption (gross national expenditure, GNE), a GDP identity, a Phillips curve, an interest rate rule and a real UIP relation for the exchange rate.

\[ \xi_t = \eta_1 \xi_{t-1} + \eta_2 E_t(\xi_{t+1}) + \eta_3 (r_t - E_t \pi_{t+1}) \] (1)

\[ y_t = a_1 \xi_t + a_2 x_t - a_3 m_t \] (2)

\[ \pi_t = \eta_4 \pi_{t-1} + \eta_5 E_t(\pi_{t+1}) + \eta_6 (y_t - y^a_t) + \eta_7 E_t \Delta \zeta_{t+1} + \eta_8 \Delta s_t + u_{Pt} \] (3)

\[ r_t = \eta_9 r_{t-1} + \eta_{10} (y_t - y^a_t) + \eta_{11} \pi_t + \eta_{12} \zeta_t + u_{rt}, \] (4)

\[ \zeta_t = E_t(\zeta_{t+1}) + (r_t - \pi_t) - (r^*_t - \pi^*_t) + u_{\zeta t} \] (5)

In these equations \( \xi_t \) is the log of absorption (GNE), \( r_t \) is a nominal interest rate, \( \pi_t \) is domestic inflation, \( y_t \) is the log of GDP, \( x_t \) is the log of exports, \( m_t \) is the log of imports, \( y^a_t \) is the potential level of output, \( \zeta_t \) is the real exchange rate, \( s_t \) is the log of the terms of trade, \( r^*_t \) is the foreign nominal interest rate and \( \pi^*_t \) is foreign inflation. If these variables are stationary they are measured as deviations from some steady state values. Later we discuss what happens if they have stochastic and deterministic permanent components. In some papers there are different datings on the expectations. Thus we might have \( E_t(\xi_{t+1}) \) and \( E_t(\pi_{t+1}) \) in place of the current values in the policy rule without changing any of the discussion. In some instances the Phillips curve has \( \pi_{t-1} \) replaced by the average inflation rate over the past four quarters e.g. Giordani (2004) and Berg et al. (2006).

In models such as Lubik and Schorfheide (2005), Justiniano and Preston (2006) and Gali and Monacelli (2005) there is no investment, so that \( \xi_t \) is consumption and the equation describing it is a standard one found from households’ optimizing behaviour. The Phillips curve is a standard open-economy one that has been derived in a number of places. Following Gali and Gertler (1999) most theoretical models of inflation have it being driven
by the deviation of marginal cost (unit labour costs) from its steady state level. If the steady state corresponds to what would happen if prices were fully flexible, than it is generally the case that one can replace that gap with the output gap, $y_t - y^*_t$, where $y^*_t$ is the level of output in a flexible price economy. Exactly what $y^*_t$ would look like then depends upon the assumptions made in deriving the model.

The GDP equation stems from a log linearization of the identity involving GDP, GNE and the trade balance. The trade balance is replaced in many models by a combination of variables such as foreign output $y^*_t$, the terms of trade and the real exchange rate. One might then write

$$a_2 x_t - a_3 m_t = \phi_3 s_t + \phi_4 \zeta_t + \phi_5 y^*_t + \varepsilon_{pt},$$

where $\varepsilon_{pt}$ is a shock reflecting changes in preferences between domestic and imported goods. The GDP equation would then be

$$y_t = a_1 \xi_t + \phi_3 s_t + \phi_4 \zeta_t + \phi_5 y^*_t + \varepsilon_{pt}$$

(6)

More often than not the GNE variable is eliminated and replaced by GDP but, as this would create quite a complex structural equation, it seems best to keep absorption and output separate.

The system above has a number of "external" variables - the levels of foreign output, interest rates and inflation are obvious ones. Exports and the terms of trade will be partly determined by internal and partly by external forces, although both are often treated as being externally determined, particularly if the country is a commodity exporter. Hence we can think of these five variables as being strongly exogenous, and they constitute observable shocks into the model. In this case solving the model results in a VARX system for the remaining internal variables, of which there are five - GNE, GDP, inflation, the domestic interest rate and the real exchange rate. It is immediately clear then that the SVARX coming from the NK model above involves the same variables as in Dungey and Pagan(2000), with the
exception that the latter has two extra variables representing external and internal real asset prices.

An issue that arises in relating empirical and theoretical models is whether the assumption of rational expectations used in theoretical models is realistic. It may be that the weights assigned to the information available when constructing rational expectations are too restrictive. Often it may make more empirical sense to work with the same information set as in theoretical models but to allow the data to determine the weights. At the same time theoretical models often regard agents as having more information than an econometrician does, and so contemporaneous information on output and inflation is taken as available when forming expectations. A different approach, which may better recognize the lack of timely information, is that the appropriate information set comprises the current observed exogenous variables, lagged values of any endogenous variable, and selected shocks. We will use this latter orientation when moving to the data.

To illustrate the argument, and derive the implications of it, we focus upon the IS curve in the NK model. It is necessary to replace the expectation of future GNE with a weighted average of variables in the model. We will assume that this expectation is a function of the observable endogenous variables at \( t - 1 \), \( z_{t-1} \), as well as current values of the observable exogenous variables, \( x_t \). For the unobservable variables we will assume that the agent knows the current permanent level of technology, \( A_t \). But, any other shocks that are not serially correlated will be set to zero when forming expectations. Consequently the expectations of future GNE are generated by

\[
E_t(\xi_{t+1}) = z'_{t-1} \lambda_1 + x'_{t-1} \lambda_2 + \psi A_t,
\]

and the IS equation takes the form

\[
\xi_t = \eta_1 \xi_{t-1} + \eta_2 (z'_{t-1} \lambda_1 + x'_{t-1} \lambda_2 + \psi A_t) + \eta_3 (r_t - E_t \pi_{t+1}).
\]

Turning to the expected level of future inflation, this would not involve the level of technology but rather its expected rate of change. If that was
white noise then the expectation would be zero and so $E_t \pi_{t+1}$ would simply be a function of $z_{t-1}$ and $x_t$, i.e. $E_t \pi_{t+1} = z'_{t-1} \gamma_1 + x'_t \gamma_2$, leading to the IS curve

$$\xi_t = z'_{t-1} \phi_1 + x_t \phi_2 + \eta_3 r_t + \psi_3 A_t.$$ 

In the event that $A_t$ has some serial correlation further lags would be introduced into the absorption equation. Similar substitutions will produce structural equations for the NK system that have lagged $z_t$ and current $x_t$ variables on the LHS. The final set of NK equations would then become

$$\xi_t = z'_{t-1} \phi_1 + x_t \phi_2 + \eta_3 r_t + \psi_3 A_t$$

(7)

$$y_t = a_1 \xi_t + \phi_3 s_t + \phi_4 \xi_t + \phi_5 y^*_t + \varepsilon_p t$$

(8)

$$\pi_t = z'_{t-1} \phi_6 + x'_t \phi_7 + \eta_6 (y_t - y^*_t) + \eta_8 \Delta s_t + u_p t$$

(9)

$$r_t = \eta_{9} r_{t-1} + \eta_{10} (y_t - y^*_t) + \eta_{11} \pi_t + \eta_{12} \xi_t + u_r t$$

(10)

$$\zeta_t = z'_{t-1} \phi_6 + x'_t \phi_7 + (r_t - \pi_t) - (r^*_t - \pi^*_t) + u_{\zeta t},$$

(11)

and we will refer to this as the adjusted NK model.

An issue arises over the real exchange rate equation. We would expect that expectations of $\zeta_{t+1}$ would be a function of the expected difference between domestic and foreign technology levels, and these may be captured by $y^*_t$ and $y_t$. If this is not so, then the shock $u_{\zeta t}$ essentially captures that differential. If foreign and domestic technology are not co-integrated then the shock in this equation would have a unit root and hence all variables would need to be expressed as differences.

### 3 Model Design with I(1) Variables

If some of the series such as output are unit root processes i.e. have permanent components, an account needs to be given of how that is to be handled in both the theoretical and empirical structures. In simple DSGE models it
is presumed that the $I(1)$ variables would be $y_t, y^n_t$ and $\xi_t$. These are rendered stationary by assuming that the log of technology $A_t$ is an $I(1)$ process, $\Delta A_t = \varepsilon_{at}$, and then defining $\tilde{\xi}_t = \xi_t - A_t, \tilde{y}_t = y_t - A_t, \tilde{y}^n_t = y^n_t - A_t$. The original variables in the above system are now replaced with the tilde values.

In the case of the IS equation and $\eta_1 + \eta_2 = 1$ this produces

$$\tilde{\xi}_t = \eta_1 \tilde{\xi}_{t-1} + \eta_2 E_t(\tilde{\xi}_{t+1}) + \eta_3 (r_t - E_t \pi_{t+1})$$

$$\Delta A_t - \eta_1 A_{t-1} + \eta_2 E_t(A_{t+1})$$

$$= \eta_1 \tilde{\xi}_{t-1} + \eta_2 E_t(\tilde{\xi}_{t+1}) + \eta_3 (r_t - E_t \pi_{t+1}) + (\eta_2 - 1) \Delta A_t.$$  

The presence of a single $I(1)$ factor $A_t$ in the system must also mean that $(\xi_t, y_t)$ and $(y_t, y^n_t)$ are co-integrating pairs, since there are three $I(1)$ variables driven by a single common permanent component. Indeed we would have the co-integrating vectors being $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, making the output gap $y_t - y^n_t$ equal the transitory component of $y_t$. This points to the need to extract the permanent components of any $I(1)$ series.

Many empirical applications of NK and DSGE models do not explicitly incorporate a process for $A_t$. Rather the data on output etc. is filtered to remove a permanent component, the filtered data is taken to be $y_t - y^n_t$, and estimation proceeds with it.$^1$ Although there can be estimation difficulties with this solution, as discussed in Fukac and Pagan (2006), if one works with a specific NK/DSGE model it may be possible to correct for this effect. However, often the filtered data is simply used in an SVAR that is loosely based on the NK/DSGE model, and this leads us to ask whether such a strategy would be satisfactory. In particular, does this model design respect the co-integrating relations in the data?

To examine this question we will assume that the $n$ variables of the

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$^1$Basically the distinction is made between supply and demand side shocks to output, where the former are viewed as permanent and the latter transitory, so that the removal of a permanent component is then regarded as removing supply side effects. Of course there may be transitory supply side shocks, and these may influence $y_t$, but to account for these requires a more complex framework.
SVAR, \( z_t \), may contain both \( I(1) \) and \( I(0) \) variables and the \( I(1) \) variables may be co-integrated. It will be convenient to assume that the data suggests that an SVECM(2) is an appropriate model - higher orders simply lead to more complex notation.

\[
B_0 \Delta z_t = \alpha^\prime \beta_{t-1} + B_1 \Delta z_{t-1} + \epsilon_t \\
\Rightarrow \Delta z_t = \alpha \beta_{t-1} + A_1 \Delta z_{t-1} + \epsilon_t.
\]

Since \( z_t \) is a mixture of \( I(1) \) and \( I(0) \) variables we need to explain the above representation. To do so suppose that there are \( r_1 \) co-integrating vectors \((\beta_1)\) among the \( I(1) \) variables, and that the structural equations corresponding to the \( n_1 I(1) \) variables are the first \( n_1 \) (in doing so we are assuming that these structural equations have been normalized so that each equation has one of the \( z_1 \) variables as the "dependent variable"). Given that, to allow for the remaining \( n - n_1 \) variables to be \( I(0) \) we simply define \( \beta \) as

\[
\beta = \begin{bmatrix} \beta_1 & 0 \\ 0 & -I_{n-n_1} \end{bmatrix}.
\]

This will mean that the equations for the \( I(0) \) variables will have \(-\alpha_j z_{j,t-1}\) appearing in the equation that has \( \Delta z_{jt} \) as its dependent variable. Consequently, \( z_{jt} \) will be \( I(0) \) provided \( |\alpha_j| < 1 \).

Now in a co-integrated system there will be \( n_1 - r_1 \) common permanent components of the form

\[
\Delta \tau_{1t} = \eta_t,
\]

where \( \eta_t \) is white noise (the permanent component of an \( I(0) \) series will be zero). In general we can write the permanent component of \( z_{1t} \), \( z_{1t}^p \), as \( z_{1t}^p = J \tau_{1t} \), with \( J = \beta_\perp (\alpha_\perp' \Gamma \beta_\perp)^{-1} \alpha_\perp' B_0^{-1} \epsilon_t \), where \( \beta' \beta_\perp = 0 \), \( \alpha' \alpha = 0 \) and \( \Gamma = I - A_1 \).\(^2\) This will mean that \( \beta' J = 0 \). It is often the case that the

\(^2\)This is of course the Beveridge-Nelson formulation of the permanent component which has the identifying assumption that \( \Delta z_t \) is white noise, in contrast to other formulations.
SVECM is then transformed to an SVAR involving \( n - r \) of the \( \Delta z_t \) and the \( r \) ECM terms \( \beta' z_{t-1} \). Examples of this are Gali (1992, 1999). However, although sometimes the ECM terms are vaguely referred to as "gaps", it is not true that they are the sorts of "gaps" that appear in NK/DSGE models. To see this we transform the SVECM to the form

\[
B_0 \Delta (z_t - z^p_t) = \alpha^* \beta' (z_{t-1} - z^p_{t-1}) + B_1 \Delta (z_{t-1} - z^p_{t-1}) + (\alpha^* \beta' + B_1) \Delta z^p_{t-1} - B_0 \Delta z^p_t + \varepsilon_t,
\]

from which it is clear that "gaps" (transitory components) are actually combinations of the ECM terms and changes in the permanent components. Indeed this decomposition makes it clear that any SVAR using gaps will also need to incorporate \( \Delta z^p_t \) in order to be compatible with the data. If \( \Delta z^p_t \) is excluded there will be extra terms entering into the error terms of the equations, and so the assumption that these are uncorrelated shocks will be invalid. The problem essentially arises due to the unobservability of the common permanent components in most theoretical models and it is the attempt to circumvent this unobservability that creates difficulties.

4 Origins of the H2007 Model

Dungey and Pagan (2000) constructed an 11 variable model of the Australian economy. Nine of these variables were those in the generic NK model outlined above - domestic inflation, the real exchange rate, exports, the terms of trade, domestic and foreign interest rates, foreign output, domestic GDP and GNE. Two extra variables appeared in the system. These measured foreign and domestic real equity prices. The argument for the latter is that GNE includes investment, and not just consumption, as in many small theoretical models, and we might therefore expect that it will be influenced by the real return such as Hodrick-Prescott that focus upon smoothness. However, it is the case that within a properly structured model it will be the BN permanent component which would be used to define the gap. If one uses HP type formulations then the analysis is quite difficult.
to capital, as measured by real equity prices.\(^3\)

In SVAR2000 the foreign variables (including the terms of trade) are modelled as a small VAR system that was exogenous to the Australian economy rather than being conditioned upon, as in SVARX systems. The argument for the latter treatment was that it enabled the discussion of the separate effects of independent shocks to exogenous variables such as US asset prices and US GDP, but that flexibility is something that is not of great interest to us here. It can always be done after the SVARX system is estimated if that is felt to be desirable. The variables in SVAR2000 also had a linear deterministic trend removed from them before analysis. This is a common strategy when theoretical models are estimated with data, in that the data is "demeaned" before estimation i.e. the means of the changes in variables are all set to zero. This is true of Lubik and Schorfheide for example. In the new model, H2007, no detrending occurred prior to estimation. Deterministic trends were included in the equations for real variables such as GNE and GDP but not for inflation, interest rates or the exchange rate.

It is worth reviewing the main differences between the SVAR2000 system and the adjusted NK model in (7)-(11). A broad difference is that the SVAR2000 model is triangular, while the adjusted NK model will only be triangular if \(\eta_3, \phi_4, \eta_{12}\) are zero i.e. there are no contemporaneous effects of the real exchange rate and interest rates on GDP, and the interest rate decision does not respond to the current value of the exchange rate. The fact that interest rates take at least one period to affect expenditure in Australia is a standard one that has been accepted for a long time, and the contemporaneous effect of the exchange rate upon interest rate decisions is always rather suspect. It was actually rejected by Lubik and Schorfheide in their analysis with Australian data. It is likely that sustained movements in the exchange rate have an effect upon interest rates, but this is more an argu-

\(^3\)Of course the real interest rate is present in the equation but this is a cost term whereas the return to investment has generally been incorporated via q-ratio theory.
ment for lagged values entering into the interest rate rule. The omission of the current exchange rate from the GDP equation is more problematic, but it is an issue of the speed of pass-through, since it represents the division of GDP between imports and domestic production as a response to the relative price change.

Turning to an examination of each of the equations in the adjusted NK model, the first domestic variable in SVAR2000 is missing from the NK system. This involves the determination of the real price of equity (OZQ) in Australia as a function of the exogenous variables, one of which is USQ. The IS equations of SVAR2000 and the adjusted NK system equations also differ in a number of ways. Since all variables in SVAR2000 were taken to be \( I(0) \), the levels of variables rather than their differences appeared in the equation for GNE. Moreover, the real US rate (RUS), USQ, USGDP and exports (EXPT) were excluded from it although AUSQ was present, as well as lagged values of many domestic variables.

The two GDP equations are the same except that SVAR2000 has a direct effect from exports, whereas in the adjusted NK model the trade balance has been substituted out. To the extent that exports are not well captured by movements in the foreign variables of SVAR2000, a separate treatment for exports may be desirable. The inflation equation in SVAR2000 assigns zero weights to USGDP, RUS, EXPT and USQ, and also utilizes a GNE rather than a GDP gap. This same gap is also used in the interest rate rule. Again the policy interest rate is assumed not to respond directly to foreign variables. Finally, the real exchange rate equation in SVAR2000 involves the same variables as in (11) but the coefficient restrictions coming from UIP are not imposed.

Thus the SVAR2000 system involves the imposition of some zero restrictions upon variables that potentially appear in the adjusted NK system (although they might also have zero weight on them in that system) and it does not impose some of the restrictions coming from the adjusted NK sys-
tem, such as UIP. Perhaps the biggest difference though is the use of a GNE rather than GDP gap. The arguments for this relate to the fact that GNE is possibly a better measure of demand in an open economy and this theme was developed more generally in Dungey and Pagan(2000).

Now, as mentioned above, no variables in SVAR2000 are regarded as having permanent components as all variables were treated as being stationary around a linear deterministic trend. In line with the propensity of most theoretical models to treat variables such as GDP as having a unit root owing to permanent technology shocks, we wish to convert SVAR2000 to an SVARX system that has such a property. Most theoretical models would treat real financial prices (after eliminating a deterministic component) as I(0), so we do this for the q ratios and interest rates. The real exchange rate is somewhat more problematic. If there are differences in technology between countries then the real exchange rate may have to adjust in order to keep the current account as a certain fraction of GDP. Based on later analysis we find that it is hard to reject the fact that there are three common permanent components in the system and, if we interpret two of these as global and local technology, the real exchange rate must have a permanent component. The ADF (3) test for whether it has a unit root is -1.47 versus a critical value of -3.46, although the point estimate of the lagged variable is only .93. Thus we will allow it to be an I(1) process. Consequently, the I(1) variables will be the logs of GDP, GNE, exports (EXPT), the real TWI (RTWI) and foreign output (measured in the paper USGDP). We test for whether there is cointegration among these five variables, treating the last two as exogenous and augmenting the tests with the I(0) external exogenous variables. The max and trace test statistics point to there being two co-integrating vectors with test statistics of 24.5 and 19.2 versus 95% critical values of 23.4 and 18.1 respectively. After imposing identifying restrictions the two vectors are

\footnote{Computations were done with MFIT5 which exploits the VARX structure in designing these tests. As in the original paper a VAR(3) was the basic model upon which tests were performed.}
(variables are in logs)

\[
GDP = 0.31USGDP + 0.07EXPT + 0.61GNE \\
GDP = 0.64GNE - 0.21RTWI
\] (13) (14)

In the last co-integrating vector we set the coefficients of USGDP and exports to zero. One of these restrictions is over-identifying and we take it to be exports. A $\chi^2(1)$ test of .01 easily accepts the constraint. A picture of the first cointegrating errors is in Figure 1.

It is interesting to observe that many small models, such as the RBA’s model of the Australian economy in Beechey et al (2000), have relied quite heavily on cointegration between Australian and US GDP, but that restriction is now regarded as untenable and is not used in the current RBA model - see Stone et al.(2005). One can see the reason for such a move by regressing the log of Australian GDP against the log of US GDP, leaving a Durbin Wat-
son test for the residuals of .23. However, if one adds into this regression the log of GNE and the log of exports it rises to .65, and a unit root in the residuals is easily rejected. The cointegrating relation above also holds over the shorter period to 1995/4 (based on the trace test), and the weights would be .57, .05 and .28, which are quite close to the estimates over the longer period. GNE clearly contains important information about permanent components.

The existence of two cointegrating vectors among the five variables means that there are three independent permanent components. This is different to theoretical models in which there is a single permanent component which is the level of technology (although there exist papers with two different components, such as Edge et al. (2007)). Most theoretical models would see domestic and foreign technology as having a common $I(1)$ component but possibly deviating from that due to some $I(0)$ factors. But the Australian data clearly rejects this idea. One possibility is that it is due to the use of data that is not in per capita form, but this is more likely to produce different deterministic trends, and that difference has already been allowed for, since we have effectively removed separate trends from each of the real variables. Another is that the lack of co-integration is a temporary phenomenon due to the large improvements in productivity in Australia during the 1990s, but, since these also seemed to occur in the US, it is difficult to fully subscribe to that argument.

Since there are two exogenous variables - foreign GDP and exports - we will treat these as two of the permanent components. From cointegration theory these would then evolve as separate $I(1)$ processes. This is often viewed as unsatisfactory, but one has to be careful to recognize that one could well have variables that don’t cointegrate, but which stay close to one another for long periods of time, unlike variables that fail to co-trend.

Exports might well have a separate permanent component for two reasons. One is that Australian markets for exports are now located mainly in the Asian region and US GDP can be a poor proxy for developments there.
Another is the well recognized phenomenon that modelling of trade flows often requires gravity models i.e. since exports and imports grow much faster than domestic (and foreign) GDP, gravity models generally relate them to a weighted average of foreign plus domestic GDP, where the weights sum to more than unity. This is certainly true of Australian export growth over much of our sample - see Figure 2 - and empirical analysis has to reflect what happens over a sample. When the model is used, however, it may be desirable to impose stronger assumptions such as co-integration and co-trending behaviour.

Given that the shocks to exports and US GDP can be used to construct two permanent components we need to be able to estimate the remaining one.\textsuperscript{5} The NK model suggests that the shock to GNE is the remaining

\textsuperscript{5}Because we treat exports and US GDP as separate univariate components the permanent component of any of the series is easily found using the univariate Beveridge-Nelson decomposition of it.
permanent shock. A secondary test of this follows the demonstration in Pagan and Pesaran (2007) that, if a structural shock is permanent, then the equation it attaches to should have no ECM term present. This was true of the GNE equation. It was not true of the GDP equation, so it seems as if the permanent shock is that from the GNE equation. It should be noted however that the adjusted NK model would point to the cointegrating relation between $y, \xi_t$ and $\zeta_t$ being given by (8), and one might therefore expect that the weights attached to $\xi_t$ would be something like one less the import share, and this might also be true of the weight attached to $\zeta_t$ if the expenditure elasticity of imports was unity. Hence these weights should be closer to -.8 and .8, but such a restriction seems to be rejected by the data. Again, this shows a tension between theoretical models that would impose such a constraint and empirical models that reflect the data, to which there seems no easy resolution.

5 From SVAR2000 to H2007

The first step in the analysis was to re-estimate SVAR2000 using an updated data set to 2006Q4. The only alteration to the specification was the inclusion of a once off dummy for the introduction of GST in the third quarter of 2000. The resulting impulse responses were in line with those reported in Dungey and Pagan (2000). This was also true of the subsequent move to SVARX2000, where the VAR model used for the international variables in SVAR2000 - USGDP, TOT, RUS, USQ and EXP - is suppressed (note that the small open economy structure of SVAR2000 meant there was no feedback from the Australian variables to the US).

In H2007 five variables are expressed in log changes - these are denoted as DUSGDP, DEXP, DGNE, DGDP and DRTWI where the D indicates the first change operator. There are two cointegrating relationships between the levels of these variables. The corresponding ECM terms are constructed ac-
according to equations (13) and (14). These ECM terms then both enter into
the DGDP equation with a single lag as this equation contains all the var-
iables from both cointegrating relationships. The second ECM term, between
GDP, GNE and RTWI, enters into the DRTWI equation. In SVAR2000 no
other equation contained all the variables in the cointegrating relationships.
For H2007 we make one change, which is to include DEXPT and its lags into
the DRTWI equation, meaning that the first ECM term also now enters this
equation - without this the convergence time of the impulses is noticeably in-
creased. The DGNE equation has each variable expressed in changes, thereby
enforcing the restriction that the shock on this equation is a permanent one.
The logic of this action is set out in Pagan and Pesaran (2007).

SVAR2000 was an SVAR(3). Estimation of H2007 was undertaken with
differenced variables having two lags for compatibility. To ensure comparabil-
ity with SVARX2000 the coefficient restrictions and structure of SVAR2000
are retained throughout. Thus the equation for DGDP contains the second
lag of CASH but omits the first one. Estimation proceeds on an equation
by equation basis just as for SVAR2000 (this was done in GAUSS 6.0). The
impulse responses are produced along the lines of the Appendix. The sizes
of the shocks administered to each model are the same in each case, cor-
responding to one standard error shocks to the equations in SVARX2000.
These are given in Table 1 (note that they are not particularly different
from those reported in Table 3 of Dungey and Pagan (2000)). Formal 90%
bootstrapped error bands for the complete set of impulse responses for the
domestic variables are provided in the Appendix.

5.1 Impulse Responses
5.1.1 Shocks to Exogeneous Variables

In a number of cases the impulse responses of a shock to a particular vari-
able are close to those from SVARX2000. An example occurs for the equi-
alent sized US GDP shock equation on OZQ in both models - see Figure
Table 1:
Size of One Standard Deviation Shocks (as percentages in SVARX2000).

<table>
<thead>
<tr>
<th>variable</th>
<th>shock</th>
<th>variable</th>
<th>shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>USGDP</td>
<td>0.6%</td>
<td>OZQ</td>
<td>7.2%</td>
</tr>
<tr>
<td>TOT</td>
<td>1.7%</td>
<td>GNE</td>
<td>0.6%</td>
</tr>
<tr>
<td>RUS</td>
<td>148bp</td>
<td>GDP</td>
<td>0.5%</td>
</tr>
<tr>
<td>EXPT</td>
<td>8.0%</td>
<td>INF</td>
<td>2.2%pa</td>
</tr>
<tr>
<td>USQ</td>
<td>3.3%</td>
<td>CASH</td>
<td>173bp</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RTWI</td>
<td>2.1%</td>
</tr>
</tbody>
</table>

3, panel (a). The fact that the shock is permanent in H2007, but transitory in SVARX2000, means that there will always be some difference as the time horizon lengthens, although the differences may not show up for some time owing to the fact that the shock may be very persistent in SVARX2000. Eventually of course there is no effect of the shocks since OZQ is assumed to be an $I(0)$ variable. Because OZQ appears directly after the foreign variables one would also expect similar impulse responses in the two models.

Of course the fact that a USGDP shock is now permanent rather than transitory will mean that there will be different effects on the $I(1)$ domestic variables in H2007, see panel (b). In the SVARX2000 model the effects of a shock to USGDP are initially larger in H2007 than SVARX2000, but the H2007 model settles to a permanent positive effect while SVARX2000 returns to its long run trend value, recrossing the baseline after almost 5 years. The effects on domestic inflation, shown in panel (c) and on interest rates (CASH) (not shown) are smaller in the H2007 model than in SVARX2000, but the changes in both are of similar magnitude, so that the real interest rate falls by a similar amount in both cases. The fall in the real interest rate prompts a fall in the RTWI in both models, panel (d), but the $I(1)$ nature of RTWI in H2007 results in a permanent depreciation.

The effects of a preference shock are seen in the terms of trade shock. In
H2007 this leads to an increase in GNE (shown in Figure 4 panel (a)) and GDP. The inflation and interest rate responses for both models are shown in panels (b) and (c). As would be expected given the nature of the relationship between the TOT and RTWI for Australia, the RTWI appreciates in response to the shock, panel (d). These effects are all transitory, although in H2007 they can take a considerable time to return to equilibrium.

A shock to RUS looks similar across the two models. The responses in the first few quarters are similar to those recorded in SVARX2000. The CASH response is faster to dissipate in the H2007 model.

The effects of a shock to USQ are consistent with those recorded in Dungey and Pagan (2000), although the shocks take longer to dissipate. The temporary increase in USQ leads to an increase in OZQ, shown in Figure 5 panel (a) and increases in each of GNE and GDP (panel (b)). The interest rate and inflation responses are quite volatile in the first 18 months, just as
in SVARX2000, but then become negative and dissipate from there, (panels (c) and (d)). The response of RTWI is larger in H2007 than in SVARX2000 (not shown).

The shock to exports is now permanent versus the transitory one of SVARX2000. The results are shown in Figure 6. A permanent increase in exports results in only a small initial rise in OZQ, panel (a). The effects of the exports shock on GDP are much greater than those on GNE - panels (b) and (c). At the end of thirty two quarters the GNE response is actually negative, possibly because the real interest rate has been held at a high level for a long time. From theoretical models it might be expected that there would only be small changes in GDP but a larger rise in GNE due to a wealth effect. The greater wealth leads to a higher expenditure on imports that matches the increased exports and that means little change in GDP. It would be possible to enforce constraints upon the long-run effects in a relatively simple way but
the current paper attempts to keep as close as possible to SVARX2000 for comparative purposes. The initial rise in GDP and GNE produces a muted response in inflation in H2007, panel (d). Domestic inflation rises to a peak of around 0.05 percent per quarter at around 2 years.

5.1.2 Shocks to Domestic Variables

This section presents results on five of the domestic shocks. The omitted shocks are those of the INF and RTWI equations. These are particularly difficult to interpret, although they might be regarded as markup and risk premium shocks. In general, the responses to both shocks are similar in the H2007 and SVARX2000 models, and they are presented in the Appendix.

OZQ shocks

The OZQ shocks have a positive impact on GNE and GDP in both the SVARX2000 and H2007 models, see Figure 7 panels (a) and (b) for example.
Figure 6: Shock to EXPTS, response in H2007 (solid line) and SVARX2000 (dashed line).
The main differences are in the scale of the responses in GDP, INF and CASH (panel (b-d)). The first two are smaller while the last is larger in the H2007 model.

The major difference between the impact of an OZQ shock on the Australian economy and that of a similarly sized exogeneous USQ shock is in the response of inflation and, consequently, CASH - see Figure 8. The USQ shock has a highly variable effect, while the response to the OZQ shock is positive for longer and reaches a peak of over 0.4 percent per quarter above baseline around one year after the shock. The responses in GDP and GNE are both positive (compare Figure 5 panel (b) with Figure 7 panels (a)). However, in the case of the USQ shock the impact lasts over 8 years for GNE and almost 6 years for GDP, while in the OZQ shock case the impact on GDP and GNE lasts some 6 and 7 years respectively.
This has been interpreted in the NK model as a domestic technology shock. It results in a strong positive effect on the levels of both GNE and GDP - see Figure 9, panels (a) and (b). The effect of the shock on the level of GNE is 1.8 times that on GDP after the first year. After 5 years GNE is 0.8 percent above baseline while GDP is 0.5 percent above it. This is consistent with a technology shock expanding incomes and inducing rises in imports. As in SVARX2000 the shock to DGNE results in a rise in inflation and CASH, see panels (c) and (d). Ideally, policy would not respond to technology shocks, but it would be very hard for a monetary authority to separate out technology shocks from demand induced rises in GNE, and so it is likely that the rise in the latter would bring forth some rise in the policy rate. In the current version of H2007 the monetary policy reaction function (represented by the CASH equation) places more emphasis on the GNE gap than inflation - the
opposite to what is reported in de Brouwer and Gilbert (2005). In H2007 the associated rise in the real cash rate is smaller than in SVARX2000 (these are shown in panel (e) and are calculated as the cash response less four times the quarterly inflation response) and the RTWI takes longer to appreciate in H2007 than in SVAR2000X, but eventually appreciates permanently, panel (f).

**GDP shocks**

The shocks to the DGDP equation, which have been interpreted as a transitory preference shift to domestic goods over imported ones, have temporary effects on GDP and GNE. As expected from a preference shock, the effect on GDP is greater than the effect on GNE - see Figure 10, panels (b)
Figure 10: Shock to GDP, response in H2007 (solid line) and SVARX2000 (dashed line)

and (c). The effect on INF is also expected to be positive, but in H2007 this is not the case initially, panel (d). The effects on CASH (panel (e)) and INF are consistent with an appreciation in the RTWI - panel (f).

**CASH shocks**

The CASH rate shocks are a broad proxy for changes in monetary policy. Here a 173 basis point shock is applied. It would be anticipated that unexpected changes in the CASH rate should result in a fall in INF, GNE and GDP. Each of these responses is evident in H2007, see Figure 11, the delays in the responses of real variables being imposed upon the models. A similar size of CASH shock in both SVARX2000 and the H2007 models results in a smaller impact on inflation in the H2007 model than the SVARX2000 model.
The rise in the real cash rate leads to an appreciation of the RTWI in both cases, panel (d). In comparison with the results here the Australian SVAR of Berkelmans (2005) reports almost a 1 percent appreciation of the RTWI in response to a 25 basis point CASH shock. To get a similar response in H2007 would require a CASH shock of 8 times the size.  

The effect of the CASH shock on GNE and GDP lasts around 7 years. There are differences in the relative magnitudes of the responses. SVARX2000 features a stronger effect of the cash rate on GNE than GDP, while in H2007 they are more similar. Moreover the effects of the cash rate shock are much weaker in H2007, see panels (a) and (b). The smaller effect in H2007 compared with SVARX2000 is presumably a consequence of the equilibrium correction mechanism in the system, and suggests that the analysis in the levels based model may overestimate the impact of monetary policy on the economy.

5.2 Historical Decomposition of Output

Historical decompositions of variables are the product of impulse responses and estimated SVAR shocks. They were one of the uses of SVAR2000 in Dungey and Pagan (2000). The appendix discusses the construction of impulses and outlines the variable decompositions used here. The decomposition of GDP into its component shocks is given in Figure 12. Each of the components is reported on the same scale, which makes it immediately obvious that the majority of the contributing shocks are sourced from technology (GNE), from preferences (GDP) and non-trivially from the foreign sector. In particular, exports have been contributing positively in more recent years. Oddly enough the terms of trade have been working against that owing to the induced strong appreciation. The positive effects of the stock market

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6Berkelmans (2005) uses a 7 variable VAR with domestic variables represented by GDP, real credit, quarterly inflation, the cash rate and RTWI. The external sector is represented by commodity prices and USGDP.
booms in the local and US markets are clearly evident in the period leading up to 2000.

To understand the impact of monetary policy more clearly, we follow Dungey and Pagan (2000) and construct a monetary policy indicator. This involves suppressing the feedback effects in the interest rate equation, and then adding the difference between those results and the equivalent ones from the original system to the pure effects of the cash rate shocks. The resulting monetary policy indicator is shown in Figure 13(a). The early part of the Figure shows quite substantial volatility, which is associated with the initial conditions rather than the model results, so it seems wisest to consider analysis from post 1986 in order to give the initial conditions time to dissipate.

The results show some differences to those in Dungey and Pagan (2000). In the earlier paper (and Dungey, 2002) the influence of monetary policy
Figure 12: Historical decomposition of GDP into its component shocks.
on output was negative from around 1987 until the end of 1993. Here this pattern is interrupted by a short-lived expansionary period for the year from September 1989 to September 1990, most likely the effects coming from looser monetary policy after the 1987 stock market crash. The earlier result that policy remained contractionary for an extended period thereafter remains in place. During the recession monetary policy was strongly expansionary, with policy tightening in 1994 and eventually becoming contractionary in September 1997. Thereafter, a more neutral effect was in place for the ensuing year. Following this, monetary policy has been expansionary for output during the remainder of the period, returning to a neutral stance towards the very end of the sample (the end of 2006).

Monetary policy over the vast majority of the sample period has been aimed at controlling inflation. Figure 13(b) conducts a similar analysis for a monetary policy indicator for inflation. The figure shows that at the end of the period, from September 2005 onwards, monetary policy has been acting to reduce inflation. For most of the sample period the effects on inflation just mirror those upon output. Reference back to Figure 12 suggests that strong domestic demand has been a major contributor to inflation at the end of the sample, aided by positive export and investment (ausq) effects.

6 Conclusion

Using our previously established empirical SVAR model of the Australian economy as a benchmark we have set out a model that explores the relationship to models which incorporate the theoretical insights from New Keynesian models. It is apparent that the New Keynesian models suggest some restrictions which are not supported by the data, and equally the SVAR suggests some restrictions which are not present in the New Keynesian models. Empirical implementations of New Keynesian models, and our previous SVAR, did not specifically account for permanent and temporary shocks in
Figure 13: Monetary Policy Indicators for Output and Inflation.

the data. Here we explored the impact of including such features into the model, allowing for a mix of I(1) and I(0) data, leading to a new model of the Australian economy, H2007. A comparison with the earlier SVAR, which included only the possibility of temporary shocks, shows that the inclusion of the longer run relationships suggests that the previous model overstated the impact of cash rate changes on macroeconomic activity, demonstrating the potential importance of these modelling innovations to policy makers.
References


A Deriving Impulse Responses and Historical Decompositions

We set out the methodology for a third order system. Let the estimated SVECM(3) system be written as

$$\Delta z_t = \alpha \beta' z_{t-1} + B_0^{-1} B_1 \Delta z_{t-1} + B_0^{-1} B_2 \Delta z_{t-2} + B_0^{-1} \varepsilon_t$$

As in the text we can write this as

$$\Delta \psi_t = \alpha \beta' \psi_{t-1} + B_1 \Delta \psi_{t-1} + B_2 \Delta \psi_{t-2} - \Delta z_t^p + B_1 \Delta z_{t-1}^p + B_2 \Delta z_{t-2}^p + B_0^{-1} \varepsilon_t,$$

where $\psi_t = z_t - z_t^p$. Therefore

$$\psi_t = \alpha \beta' \psi_{t-1} + B_1 \Delta \psi_{t-1} B_2 \Delta \psi_{t-2} - J \varepsilon_t + B_1 J \varepsilon_{t-1} + B_2 J \varepsilon_{t-2} + B_0^{-1} \varepsilon_t$$

since $\Delta z_t^p = J \varepsilon_t$, where $J = \beta_\perp (\alpha'_\perp \Gamma \beta_\perp)^{-1} \alpha'_\perp B_0^{-1}$. This provides a VARMA system in the $I(0)$ variables $\psi_t$ which can be written as

$$G(L) \psi_t = H(L) \varepsilon_t$$

showing that the impulse responses are $G(L)^{-1} H(L)$. These can be computed in the normal way for a stationary system.

Once we have the impulse responses $\frac{\partial \psi_t + \varepsilon_t}{\partial \varepsilon_t}$, we have

$$\frac{\partial z_{t+1}}{\partial \varepsilon_t} = \frac{\partial \psi_{t+1}}{\partial \varepsilon_t} + \frac{\partial z_{t+1}^p}{\partial \varepsilon_t}$$

$$= \frac{\partial \psi_{t+1}}{\partial \varepsilon_t} + J.$$

Historical decompositions can be derived by simply recognizing that the VMA form allows for any variable to be written as a weighted sum of previous shocks, with an initial condition, such that,

$$\Delta z_t = \text{initial conditions} + \sum_{i=0}^{t} C_i \varepsilon_{t-i} + J,$$

where $C_i$ represent the impulse response functions.
A.1 Confidence Intervals for Impulse Response Functions

For completeness, bootstrapped 90% error bands for H2007 from 5000 bootstraps were conducted in Gauss 6.0 using the rndu generator. These are presented for the effects of each shock on the domestic variables in the model. The impulse responses presented in the paper are scaled so that the SVARX and H2007 shocks are the same size.