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Econometric Analysis and Prediction of Recurrent Events

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Abstract

Economic events such as expansions and recessions in economic activity, bull and bear markets in stock prices and financial crises have long attracted substantial interest. In recent times there has been a focus upon predicting the events and constructing Early Warning Systems of them. Econometric analysis of such recurrent events is however in its infancy. One can represent the events as a set of binary indicators. However they are different to the binary random variables studied in micro-econometrics, being constructed from some (possibly) continuous data. The lecture discusses what difference this makes to their econometric analysis. It sets out a framework which deals with how the binary variables are constructed, what an appropriate estimation procedure would be, and the implications for the prediction of them. An example based on Turkish business cycles is used throughout the lecture.

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1 Introduction

There are many cases where macroeconomic and financial variables seem to exhibit *recurrent* events of one form or another. These events pertain to patterns in a series y_t . To describe them one utilizes a binary random variable S_t , taking the values of unity and zero. Examples would include:

1. Cycles in economic activity. Here a series y_t is chosen to represent economic activity and a cycle in it involves phases of expansions, $S_t = 1$, and contractions, $S_t = 0$. If the series y_t represents the *level* of economic activity then it is the *business cycle* that is being isolated. If a permanent component is taken away from y_t we are investigating the *growth cycle*.
2. Bull and bear markets. The underlying variable here will be some asset price e.g. the Dow-Jones or the S&P500. More recently there has been an increasing interest in *financial cycles* and a recent review by Claessens et al (2011)) utilizes data to determine phases in equity prices, house prices and credit. This research has been stimulated by interest in exploring the interactions between financial and real cycles.
3. Financial crises. Here a unity indicates that a crisis is occurring while a zero indicates that this is not a crisis period. There are a variety of types of crises. Bordo et al. (2009) distinguish Banking, Foreign Currency and Debt crises, as well as what is referred to as "sudden stops" caused by the sudden withdrawal of capital by foreigners. The binary variables for these are constructed in a variety of ways. Thus, a banking crisis involves the level of bank capital, while a foreign currency crisis generally involves the size of movements in exchange rates and international reserve changes - see Eichengreen et al. (1985) and Kaminsky et al. (1998). To identify a "sudden stop" event Bordo et al. use either an annual decline in capital inflows of more than two standard deviations away from the mean annual growth rate or a decline exceeding a predetermined fraction of GDP. To these are added the contingency that GDP must decline either in the year of the sudden stop or the year after. In the Early Warning System literature it is often the case that researchers work with the crisis data constructed by Kaminsky et al (1998) who proposed a pressure index which combined together the changes in interest rates and reserves. Lestano and Jacobs (2004) later modified this to include the change in exchange rates.
4. IPO markets are often classified as hot ($S_t = 1$) and cold ($S_t = 0$) depending upon either the volume of new offers or the behaviour of excess returns - see Ibbotson and Jaffe (1975) and Brailsford et al. (2001).

5. Commodity and real estate markets are often classified as booms and slumps depending upon movements in the respective underlying prices e.g. Cashin et al. (2002).
6. There are a variety of other applications in which binary random variables are involved e.g. Hausmann et al. (2005) defined periods of "growth accelerations" as those in which there was an increase in per capita growth by more than 2 percentage points, and then analysed what determined the binary variable that resulted (there were other constraints - growth had to be at least 3.5% after the acceleration and output in the acceleration phase had to exceed what it was before that).

We notice from this description that patterns in a series y_t are being investigated and that the S_t are constructed from such series, We might ask why we would summarize the recurrent events in this way? Why not just utilize the y_t directly rather than working with some summary of its behaviour like S_t ? Here are some answers that have been provided.

1. The S_t may be chosen to emphasize some feature in y_t that is not immediately obvious. This is a common device in econometric research. Thus squaring the data loses information on the sign but emphasizes volatility. In a similar way binary random variables can focus attention on the frequency and length of times spent in states $S_t = 1$ and $S_t = 0$.
2. The second is to reduce the dimension of the data generating process so as to more easily discern hidden patterns in the data or to isolate characteristics that a model seeking to interpret the data would need to incorporate. From standard econometric analysis one might cite the decomposition of y_t into its permanent and transitory components, as this becomes a key step in economic model design. Often when the binary variables are interacted with the y_t the resulting variables can point to important characteristics in y_t that economic models need to account for. A well studied example is that some U.S. business cycle expansions are not smooth, but generally feature a period of fast growth. This has also been observed in bull markets as well - see Pagan and Sussounov (2003).
3. Meaningful to decision makers. Because of the well documented phenomenon of loss aversion it is probably not surprising that decision

makers are very sensitive to whether there will be a contraction in series such as GDP and the S&P500. Reactions to such an event from the electorate or clients are often very strong. Consequently, this has led to great interest in being able to predict such events and to examine their causes. This motivates why one might wish to determine (say) the Data Generating Process (DGP) of the S_t representing contractions and expansions, given a known DGP for y_t . In these instances one is particularly interested in whether one can predict S_{t+1} given information available at t . Thus predicting recessions has long been of interest, while in recent times proposals for "early warning systems" of financial crises have emerged.

4. Often a number of S_t are constructed and used to study questions such as whether cycles are synchronized across sectors or countries. Moreover, it is often the case that any particular S_t is constructed from a number of series and so they represent a succinct way of examining such questions. Perhaps the best known example is those who follow the NBER methodology to construct business cycle states S_t for different countries. Often these utilize a number of series when determining the month that a recession or expansion occurred. In such cases it is more convenient to examine the coherence of the cycles, as measured by their representative S_t , then to try to find correlations between the underlying series that they might have been derived from.
5. Sometimes there may be large short lived movements in Δy_t that can affect statistics based upon the latter, but which have little effect upon the constructed S_t e.g. the stock market crash of October 1987 and the decline in output during the Great Depression. In this instance one might wish to obtain a more robust measure of some feature using the S_t rather than the Δy_t .

The above establishes that there is a large applied literature which constructs and utilizes binary variables S_t . In this lecture we will ask what econometric issues arise when one tries to utilize the S_t . It might be thought that there is little that is special as micro-econometrics handles binary random variables all the time. But those random variables are generally not constructed from some observable underlying variable y_t , and the construction process turns out to be crucial to how one should handle the S_t . For

this reason we devote the next section to describing how they are typically constructed.

2 Constructing Measures of Recurrent States and Their Nature

Often the user of the S_t is not the producer. Consequently, the researcher may just have a set of binary data S_t available and (sometimes) knowledge of the y_t they have been constructed from. To understand the nature of the S_t we therefore need to have some idea of the transformations that convert y_t into S_t . Although we may not know precisely how this is done, in most instances enough information is provided along with the data on the S_t to enable a good approximation to it. It is worth thinking of the conversion process from y_t to S_t as involving three stages, and to see how the nature (DGP) of S_t changes at each stage. We do this in the subsequent sub-sections.

2.1 Stage 1: Formulating State Change Rules and Their Effects

In the first stage we seek to determine what state the system is in at various points in the sample path. In the business cycle context, where we are seeking states of expansion and contraction, it is often the case that these are identified by locating the *turning points* in the series y_t . Consequently, these first stage turning points are produced by a set of *rules* to do that, being formalized in algorithms such as that due to Bry and Boschan (BB)(1971) and a simplified quarterly version of it (BBQ) described in Harding and Pagan (2002). In other cases rules are found by using the output from fitting statistical models such as latent Markov Processes to the y_t series — Hamilton (1989). In all instances these rules transform y_t into S_t .

Because turning point rules are widely used in the analysis of business cycles (and are the basis of the Turkish business cycle data that we utilize later for empirical work) we often focus on them in what follows. Turning points are found by locating the local maxima and minima in the series y_t . A variety of rules appear in the literature to produce the turning points. We note that a rule describing a turning point is also a *state termination rule* i.e. it indicates what initiates a change in phase, and it is sometimes

useful to think in terms of this latter perspective. It will be helpful to study three of these rules in order to understand how each influences the nature of the univariate DGP for S_t , and to understand the resulting inter-relations between S_t and any regressors x_t that are thought to influence y_t and the state S_t . The impact of any given rule will depend upon the DGP of y_t . Consequently, we will study how the mapping between y_t and S_t changes as we modify either the rules or the DGP of y_t .

2.1.1 Calculus rule

The simplest method for locating turning points is what might be termed the *calculus rule*. This says that a *peak* in a series, y_t , occurs at time t if $\Delta y_t > 0$ and $\Delta y_{t+1} < 0$.¹ The reason for the name is the result in calculus that identifies a maximum with a change in sign of the first derivative from being positive to negative. A *trough* (or local minimum) can be found using the outcomes $\Delta y_t < 0$ and $\Delta y_{t+1} > 0$. In this case the states S_t are defined as $S_t = 1(\Delta y_t > 0)$, so that S_t depends only on contemporaneous information. Note that we could have formulated the rule as $S_t = 1(\Delta y_t > 0 | S_{t-1} = \{0, 1\})$, in which case it describes how the state changes. In that guise it might be called a termination rule, although the past state is effectively irrelevant. This rule has been popular for defining turning points in economic activity when y_t is yearly data - see Cashin and McDermott (2002) and Neftci (1984). A variant of it that is extensively used in the Early Warning Systems literature is to form $1[\sum_{j=1}^n ((\Delta \ln z_{jt} - \mu_j)/\sigma_j) - k_j > 0]$, where z_{jt} are series such as the log of the exchange rate and foreign reserves, μ_j and σ_j are their means and standard deviations, and k_j are pre-defined constants.

The DGP of S_t and its Modelling Suppose that Δy_t is a Gaussian covariance stationary process and the calculus rule is employed. In this instance, Kede(1980, p34) sets out the relation between the autocorrelations of the Δy_t and $S(t)$ processes. Letting $\rho_{\Delta y}(k) = \text{corr}(\Delta y_t, \Delta y_{t-k})$, and $\rho_S(k) = \text{corr}(S_t, S_{t-k})$, he determines that

$$\rho_S(k) = \frac{2}{\pi} \arcsin(\rho_{\Delta y}(k)). \quad (1)$$

¹Since turning points are invariant to monotonic transforms of the data it is best to treat y_t as being the log of a variable such as activity.

Thus $\text{corr}(S_t, S_{t-k}) = 0$ only if $\text{corr}(\Delta y_t, \Delta y_{t-k}) = 0$. Notice that the order of the S_t process changes with the degree of serial correlation in the Δy_t series. If the series is non-Gaussian, as would be true of those involving financial variables, there may be serial correlation in S_t , even if there is none in Δy_t . Thus series such as stock returns which have little serial correlation could nevertheless have quite large amounts in their sign ($1(\Delta y_t > 0)$) owing to the presence of GARCH in the returns.

Suppose the underlying process for Δy_t is

$$\Delta y_t = x_t' \theta + \varepsilon_t, \quad (2)$$

where x_t is assumed to be strictly exogenous (and so can be conditioned upon) and ε_t is *n.i.d.*(0, 1). Then $S_t = 1(x_t' \theta + \varepsilon_t > 0)$ and a static Probit model would clearly capture the relation between S_t and the single index $z_t = x_t' \theta$, since $\Pr(S_t = 1 | z_t, S_{t-1}) = \Phi(z_t)$.

2.1.2 Two-quarters rule

The rule that two quarters of negative growth terminates a recession is often cited in the media. This is the first of the rules to prescribe what it is that terminates a state i.e. it describes the transition probability from one state to another. Extended symmetrically, so that the beginning of an expansion is signalled when there are two successive quarters of positive growth, produces the “two-quarters rule”:

$$\begin{aligned} S_t &= 1 \text{ if } (\Delta y_t > 0, \Delta y_{t+1} > 0 | S_{t-1} = 0). \\ S_t &= 0 \text{ if } 1(\Delta y_t < 0, \Delta y_{t+1} < 0 | S_{t-1} = 1) \\ S_t &= S_{t-1} \text{ otherwise.} \end{aligned} \quad (3)$$

For later reference it is instructive to write the rule as

$$S_t = S_{t-1} - S_{t-1} \wedge_{t-1} + (1 - S_{t-1}) \vee_{t-1} \quad (4)$$

where \wedge_t is a binary variable taking the value unity if a peak occurs at t and zero otherwise, while \vee_t indicates a trough. By definition $\wedge_t = S_t(1 - S_{t+1})$ and $\vee_t = S_{t+1}(1 - S_t)$ and, in the two-quarters rule case,

$$\begin{aligned} \wedge_t &= 1(\Delta y_{t+1} < 0, \Delta y_{t+2} < 0) \\ \vee_t &= 1(\Delta y_{t+1} > 0, \Delta y_{t+2} > 0) \end{aligned}$$

Lunde and Timmermann (2004) and Ibbotson and Jaffee (1975) used variants of this non-parametric rule for finding bull and bear periods in stock prices and hot and cold markets for IPO's respectively. The latter defined a hot market as being signalled by whether excess returns and their changes for two periods exceed the median values. Eichengreen et al. (1995) and Classens et al (2008) employ rules of this type to establish the location of crises in time.

The DGP of S_t and its Modelling It is clear from (4) that the S_t will have serial correlation. Suppose that Δy_t was generated as

$$\Delta y_t = x_t' \beta + \sigma e_t, \quad (5)$$

where e_t is *n.i.d*(0, 1). Then the DGP of S_t would be

$$\Pr(S_t = 1 | S_{t-1}) = S_{t-1} - S_{t-1} E[1(\Delta y_t < 0, \Delta y_{t+1} < 0) | x_t] + (1 - S_{t-1}) E[1(\Delta y_t > 0, \Delta y_{t+1} > 0) | x_t].$$

The model for S_t is no longer a simple Probit model owing to the dual index. Serial dependence in S_t will now be present caused by the fact that the probability depends on the previous state. There may of course be extra serial correlation induced in S_t by the nature of \wedge_t and \vee_t .

2.1.3 Bry-Boschan Type and BBQ rules

Neither the calculus rule nor the ‘‘two quarters’’ rule accurately describes the underlying rule used by the NBER's approach to locating local peaks and troughs in y_t . To match the features of the NBER-constructed S_t requires that a local peak in y_t occurs at time t if $y_t > y_s$, for s in a window $t - k < s < t + k$. with a trough being defined in a similar way. By making k large enough we also capture the idea that the level of activity has declined (or increased) in a sustained way. This rule, with $k = 5$ months, is the basis of the NBER business cycle dating procedures summarized in the Bry and Boschan (1971) dating algorithm. The comparable BBQ rule sets $k = 2$ for quarterly data. Now the formula given above for the evolution of S_t under the two-quarters rule continues to hold but with different driving forces due to a new definition of peaks and troughs. Thus, in the case of BBQ rules

$$\begin{aligned} \wedge_t &= 1(\Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0) \\ \vee_t &= 1(\Delta y_{t+1} > 0, \Delta_2 y_{t+2} > 0). \end{aligned}$$

These turning point rules have been used in other contexts than the business cycle e.g. the dating of bull and bear markets in monthly equity prices by Pagan and Sussonov (2003), Bordo and Wheelock (2006) and Claessens et al (2008). For monthly asset prices however one generally sets $k = 8$ or something higher, since the higher volatility of monthly asset prices means one generally wants a longer period of decline before one would be confident about the emergence of a bear market.

2.1.4 Markov Switching Rules

An alternative way of formulating a turning point rule is to base it upon the output of some model for Δy_t . By far the most popular of these has been the Hidden Layer Markov Chain, introduced into econometrics by Hamilton (1979). This is often given the shortened descriptor of a Markov Switching (MS) model, with the simplest variant having the form

$$\Delta y_t = \mu_t + \beta \Delta y_{t-1} + \sigma \varepsilon_t \quad (6)$$

$$\mu_t = \mu_1 \xi_t + (1 - \xi_t) \mu_0 \quad (7)$$

$$p_{ij} = \Pr(\xi_t = i | \xi_{t-1} = j), \quad (8)$$

where ξ_t is a binary random variable that follows a first order Markov process with transition probabilities p_{ij} and ε_t is *n.i.d*(0, 1).

Now the dating rule employed to find states involves comparing $\Pr(\xi_t = 1 | F_t)$ (where F_t is some observed data on Δy_t and its past and/or future history), with a critical threshold value c . Often $c = .5$.² This comparison produces a new binary random variable of the form $\zeta_t = 1([\Pr(\xi_t = 1 | F_t) - c > 0])$. Then, when $\zeta_t = 1$, we would be in a bull market/crisis etc and, if zero, we wouldn't be. *It is crucial to note that $\zeta_t \neq \xi_t$.* ζ_t is directly comparable with the S_t described in the preceding sub-sections. It is the ζ_t that are the recurrent states of interest, not ξ_t , although this does not seem well understood in MS applications, where one often sees ξ_t described as

²There have been many other values used though. Chauvet and Morais (2010) for example set c equal to the sample mean of $\Pr(\xi_t = 1 | F_t)$ plus one standard deviation. Candelon et al. (2010) consider a range of methods to determine an optimal c which are various functions of the Type I and Type 2 errors which would occur for any given value of c . Of these the most common method is to choose c to minimize the ratio of false to true positive outcomes. Clearly such a criterion relies upon the fact that we have already established a "true" set of indicators.

recession and expansion states etc., when they are not that at all. Indeed the properties of ζ_t and ξ_t can be different. For example Maheu and McCurdy (2000) fitted a Markov Switching model with duration dependence in the latent states to a series on U.S. equity returns (this is the model DDMS-DD in their Table 4). Based on $\Pr(\xi_t = 1|F_t)$ Maheu and McCurdy state that 90% of the time the market is in a bull state. However, if one looked for turning points in the level of equity prices, we find that $\Pr(S_t = 1) = .7$, and so bull markets hold just 70% of the time.

We note that the rule $\zeta_t = 1([\Pr(\xi_t = 1|F_t) - c] > 0)$ is like the calculus rule and makes no reference to what the previous state would be. So serial correlation in the S_t would come from the behaviour of $\Pr(\xi_t = 1|F_t)$. Harding and Pagan (2003) observed that one might approximate the MS model in (6)-(8) by a state space form, and then $\Pr(\xi_t = 1|F_t)$ would be generated by a Kalman filter i.e. there would be an autoregressive equation producing it. That would then induce serial correlation into the S_t . As for the calculus rule it would be complex.

Often the MS model is "validated" by comparing ζ_t with the S_t coming from the methods discussed in previous sub-sections e.g. Hamilton compared the ζ_t he constructed with the S_t of the NBER. Although these were a good correspondence, others have found that basic MS models give a much less satisfactory match when applying the model to other countries and series. This has led to increasingly complex MS models. Thus the AR coefficients β and σ have been allowed to shift according to the latent state outcomes, while the transition probabilities have been made to depend upon predetermined variables and the duration of the phases (Filardo (1994). More recently the latter have been allowed to be stochastic e.g. by Billio and Casarin (2010).

One issue with MS models is that there can be computational problems in finding an optimum to the likelihood (or computing the posterior). As Smith and Summers (2004, p2) say " These models are globally unidentified, since a re-labelling of the unobserved states and state dependent parameters results in an unchanged likelihood function". Applying MCMC type methods doesn't resolve it as the labelling problem means that one is drawing from different densities and this can affect convergence of the sampler. The labelling issue has been discussed a good deal in statistics and a number of proposals have been made to deal with it e.g. Frühwirth-Schnatter (2001) but few of these seem to have been applied to empirical work with MS models in economics.

As an example we fitted the MS model in (6)-(8) using the data on Turk-

ish growth rates over 1988:1-2010:1 that are used later in the paper. *Prima facie* the parameter values seem reasonable, given that they imply expansions lasting 11.43 quarters and recessions 4.7, as these are not too far from those of 12.1 and 3.5 given in Pagan (2010) using the BBQ turning point rules. But a closer examination of the fitted model properties reveal a prediction that 61% of growth rates would be negative, whereas over this sample period it is just 26%. The most likely explanation for this poor fit would be difficulties in maximizing the likelihood referred to above. It seems likely that this identification problem will be increasingly an issue as the MS models become more complex. Thus Frühwirth-Schnatter comments in connection with a three state model estimated by Chib (1996) that " although the choice of prior means obviously implies the belief that the first state has the lowest mean and the last state highest mean, the prior does not prevent label switching....rendering the parameter estimates in Chib (1996) somewhat dubious" (p205-206). Indeed, there are major differences between Chib's estimates (Table 3 p206) and hers, which were based on her preferred solution method of "permutation sampling".

What we have with MS rules is a model for Δy_t and a dating rule for isolating when we are in certain states. In contrast the rules described in previous sub-sections did not specify a model for Δy_t . It is true that the *effects* of the rules will depend upon the nature of Δy_t , but there seem to us advantages in separating these two activities. This is particularly true given that MS modelers will often validate their model (and hence dating rule) by reference to the fact that ζ_t is close to an S_t constructed by (say) the NBER i.e. some S_t is taken as the data summary to be reproduced. There are also many exercises which require that crises, recessions etc be identified by a dating rule that is independent of a particular model, such as an MS one. For example suppose one wishes to see whether an RBC model with technology shocks driven by an AR(1) process would produce recessions such as we observe in practice. Then output growth from such a model does not follow an MS process. Are we to then say that no business cycle can be generated by such a model? Therefore, it does not seem productive that one insist on data being represented by a particular statistical model. Moreover it is not clear that an MS model would be the best way of accounting for any non-linearity in output growth. There are many other non-linear models that could be used e.g. de Jong et al. (2005). Sometimes the justification used for the MS strategy is that one needs an MS model to take account of past state information when predicting future states. As we will show later

this is incorrect - it is the *definition of the state* which provides the structure needed to do the prediction.

2.2 Stage 2: Imposing Persistence on the Recurrent States

The second stage in constructing S_t from y_t involves selecting points in time when phases change in such a way as to satisfy certain requirements relating to minimum completed phase lengths. In the case of recessions and expansions we know that it is a standard requirement of the NBER when dating business cycles that completed phases have a duration of at least five months. This requirement is evident in the NBER business cycle data, where there is no completed phase with duration of less than five months. By default this seems to have been used around the world by others when dating the business cycle. Thus Chauvet and Morais (2010), when dating recessions in Brazil using an MS model, say "We augment this definition with a rule specifying how long a phase must persist before a turning point is identified". Failure to implement such a constraint can lead to odd results. For example, Chen (2005) uses an MS model to find dates for bull and bear stock markets. For nominal returns one of his bull markets lasts only a month and for real returns just two months. This is clearly unsatisfactory, but often occurs with MS models used to date phases. Of course, one could impose a minimum phase constraint by passing the turning points produced by application of an MS program through a program designed to impose the constraints, something we now turn to.

The minimum phase constraint is evident in many data series on S_t . For example it is noticeable that, for many S_t representing financial crises, there appears to be a minimum duration of time spent in each of the two states, and this is greater than a single period. This seems likely to be the outcome of imposing the belief just described that states should persist for some time if they are to be of interest to decision makers. We will refer to such constraints as *first level censoring*. In practice censoring can involve more than just the duration of the phases and may involve a *second level of censoring* e.g. we might wish to study extreme variants of bull and bear markets as Bordo et al. (2009) do. They define *booms* as bull markets that either have a duration of 3 years and an annual rise in the real stock price of 10% or which last 2 years with an annual rise of 20%. *Busts* last at least 12

months and feature an annual decline of at least 20%. So booms will be a sub-set of bull markets and busts a sub-set of bear markets. The remaining bull and bear markets get classified as "normal". So in this case we have a trivariate set of binary indicators which relate to booms, busts and normal periods. Notice that the duration censoring employed by Bordo et al. (2009) is asymmetric. With level two censoring it becomes very difficult to know precisely how this changes the DGP of the binary variables, but it is worth seeing what the impact of level one censoring is, and this is done in what follows.

Now the restriction that states must last two periods enforces some constraints upon regressions using them. Suppose we fit a regression of the form

$$E(S_t|S_{t-1}, S_{t-2}) = \alpha_0 + \alpha_1 S_{t-1} + \alpha_2 S_{t-2} + \alpha_3 S_{t-1} S_{t-2}. \quad (9)$$

An observed pattern $\{S_{t-1} = 1, S_{t-2} = 1\}$ (expansion) is compatible with S_t being either 1 or 0. But the pattern $S_{t-1} = 0, S_{t-2} = 1$ can only result in $S_t = 0$, since it would be impossible for S_{t-1} to be a contraction, as $(1, 0, 1)$ would mean a contraction which lasted only a single period. Similarly $S_{t-1} = 1, S_{t-2} = 0$ must be followed by $S_t = 1$. Hence, imposing these constraints on (9), means that

$$\alpha_0 + \alpha_2 = 0 \quad (10)$$

$$\alpha_0 + \alpha_1 = 1. \quad (11)$$

Using the quarterly S_t presented on the NBER web page for 1959/1 to 1995/2 the OLS regression in (9) of S_t on a constant, S_{t-1} , S_{t-2} , and $S_{t-1}S_{t-2}$ is performed (Newey-West (1987) HAC t-ratios in brackets for window-width of four periods), giving

$$S_t = \underset{(3.8)}{0.4} + \underset{(5.6)}{0.6S_{t-1}} - \underset{(-3.8)}{0.4S_{t-2}} + \underset{(3.1)}{0.35S_{t-1}S_{t-2}} + \eta_t. \quad (12)$$

It is clear that the expected restrictions eventuate and these are simply due to the phase length restrictions and do not arise from the nature of the data Δy_t .

For BBQ rules, which ensure that the states alternate and which impose a minimum duration of two quarters to be spent in each state, Harding (2010)

establishes that the S_t follow the following recursion (for quarterly data)³

$$S_t = S_{t-1}(1 - S_{t-2}) + S_{t-1}S_{t-2}(1 - \wedge_{t-1}) + (1 - S_{t-1})(1 - S_{t-2})\vee_{t-1}, \quad (13)$$

where \wedge_t and \vee_t again indicate peaks and troughs respectively. In BBQ

$$\begin{aligned} \wedge_t &= 1(\{\Delta y_t > 0, \Delta_2 y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\}) \\ \vee_t &= 1(\{\Delta y_t < 0, \Delta_2 y_t < 0, \Delta y_{t+1} > 0, \Delta_2 y_{t+2} > 0\}). \end{aligned}$$

Why should we have the form (13)? This can be seen by enumerating the possible outcomes. Thus $\{S_{t-2}, S_{t-1}\} = \{0, 1\}$ must mean $S_t = 1$ while $\{S_{t-2}, S_{t-1}\} = \{1, 0\}$ means $S_t = 0$. The other combinations do not yield exactly predictable outcomes and will depend on the outcomes of the binary variables \wedge_t and \vee_t . By comparing (13) with (4) it can be seen that the dependence in S_t has lengthened to be at least of second order. It should be noted that (13) can be used to impose a phase constraint on the output (say) from an MS model. One would replace \wedge_{t-1} by $(1 - \zeta_t)(1 - \zeta_{t+1})$ and \vee_t by $\zeta_t \zeta_{t+1}$, where the ζ_t are the binary states coming from the MS procedure.

If we now consider some F_t containing the information available at t (and which is assumed to include S_{t-1} and S_{t-2} here) we can write

$$\begin{aligned} E(S_t|F_t) &= S_{t-1}(1 - S_{t-2}) + S_{t-1}S_{t-2}E[(1 - 1\{\Delta y_t < 0, \Delta_2 y_{t+1} < 0\})|F_t] \\ &\quad + (1 - S_{t-1})(1 - S_{t-2})E[1\{\Delta y_t > 0, \Delta_2 y_{t+1} > 0\}|F_t], \end{aligned}$$

showing that to fit a model to the S_t we need to allow for a second order process at a minimum (due to S_{t-1}, S_{t-2}), but there could be further dependence in S_t coming from the conditional expectations $E[1\{\Delta y_t < 0, \Delta_2 y_{t+1} < 0\}|F_t]$ and $E[1\{\Delta y_t > 0, \Delta_2 y_{t+1} > 0\}|F_t]$. Thus the nature of the DGP varies according to the censoring rule and the process followed by the variable the states are constructed from. It is clear that this makes whatever model we choose to capture $E(S_t|F_t)$ have a *double index* form.

³There is a small complication caused by completed cycles having a minimum duration of five quarters which is meant to emulate the NBER constraint that phases must be at least fifteen months long. Only occasionally does this constraint bite.

In the case of monthly data imposing a minimum duration of five months for a phase would result in

$$\begin{aligned}
S_t = & S_{t-1}(1 - S_{t-2}) + S_{t-1}S_{t-2}(1 - S_{t-3}) + S_{t-1}S_{t-2}S_{t-3}(1 - S_{t-4}) \\
& + S_{t-1}S_{t-2}S_{t-3}S_{t-4}(1 - S_{t-5}) \\
& + S_{t-1}S_{t-2}S_{t-3}S_{t-4}S_{t-5}(1 - \wedge_{t-1}) \\
& + (1 - S_{t-1})(1 - S_{t-2})(1 - S_{t-3})(1 - S_{t-4})(1 - S_{t-5}) \vee_{t-1}.
\end{aligned}$$

To motivate the form observe that a sequence for $\{S_{t-5}, S_{t-4}, \dots, S_{t-1}\}$ of $\{0, 1, 1, 1, 1\}$ must mean $S_t = 1$, while $\{1, 0, 0, 0, 0\}$ implies $S_t = 0$. Accordingly, there would again be restrictions on the coefficients of whatever model was fitted to S_t . Moreover, it is clear that the process must be at least a fifth order autoregression and, when using an NBER-type monthly dating rule,

$$\begin{aligned}
E(S_t|F_t) = & S_{t-1}(1 - S_{t-2}) + S_{t-1}S_{t-2}(1 - S_{t-3}) + S_{t-1}S_{t-2}S_{t-3}(1 - S_{t-4}) \\
& + S_{t-1}S_{t-2}S_{t-3}S_{t-4}(1 - S_{t-5}) \\
& + S_{t-1}S_{t-2}S_{t-3}S_{t-4}S_{t-5}(1 - E[1\{\Delta_j y_{t-j} < 0\}_{j=0}^4|F_t])) \\
& + (1 - S_{t-1})(1 - S_{t-2})(1 - S_{t-3})(1 - S_{t-4}) \\
(1 - S_{t-5})E[1\{\Delta_j y_{t-j} > 0\}_{j=0}^4|F_t]). & \tag{14}
\end{aligned}$$

2.3 Stage 3: Judgement

Although there are exceptions, in many instances the S_t researchers are presented with involve modifying the S_t that would one would get from the two stages above. This modification stems from the application of expert judgement or what has often been called the "narrative approach".⁴ It should be emphasized that there is no doubt that the two stages above are inputs into the final decision. Because of this, the lessons learned from the analysis presented above are important when working with the final S_t . In particular, the nature of the process for S_t established in stages one and two is likely to carry over to the final states. This is evident from (12), where the S_t used in the regression are the final states selected by the NBER Dating Committee.

It has also been found that there is a close correspondence between the published NBER S_t and those coming from an application of the BB and

⁴When business cycle dating first began a key source for deciding on final dates was Thorp(1926).

BBQ algorithms. In many ways the situation is like a "Taylor rule" for describing interest rate decisions. The FOMC do not use a linear Taylor rule but it is often a good description of their behavior. But one should be wary of assuming that it is a precise description. It may be that the information in the Taylor rule maps into the decision in a non-linear way or with a different lag structure. Thus one needs to be flexible in choosing a model for S_t . When seeking general representations of binary time series it is natural to apply the folk theorem (see Meyn 2007, p538) that "every process is (almost) Markov". In our context this would mean that S_t will follow processes like (12), which we will term the Markov process of order two (MP(2)). Higher order MP's would involve higher order lags and cross products between the lagged values. Because these MP processes are effectively non-linear autoregressions they can approximate processes such as Startz's (2008) (Non-Markov) Binary ARMA (BARMA) model to an arbitrary degree of accuracy provided they are of sufficiently high order. Just as VAR's are mostly preferred to VARMA processes in empirical work due to their ease of implementation, we feel that Markov processes should be the work horse when modelling constructed binary time series.

3 Estimating Models with Constructed Binary Data

The estimation problem has two dimensions to it.

1. There will be equations where the binary variables S_t (and lags) will be the dependent variable or a regressor. These will be called the equations of interest.
2. There will be an equation (or equations) describing how S_t evolves. These will be called the state dynamics equations.

Each of these poses different problems and the answers in the literature have often not been sufficiently attentive to the fact that the binary variables are constructed from other variables.

3.1 The Equations of Interest

At a basic level one might be interested in using S_t (or its lags) as a determinant of a variable z_t i.e. we might fit relations such as

$$z_t = a + bx_t + cS_t + dx_tS_t + e_t, \quad (15)$$

where the effect of x_t upon z_t may change according to the outcome of the binary variable. An example would be when z_t is output, x_t is an interest rate, and S_t describes the cycle. Such possibilities are often mentioned. In particular there have been tests for the asymmetric effects of monetary policy e.g. Cover (1992), but in the past these tests have been done through a definition like $S_t = 1(w_t > 0)$, where w_t has mostly been e_t . Clearly such tests do not effectively address the question of whether the impact of monetary policy is different in different phases of the cycle, since an $S_t = 1(w_t > 0)$ does not match published business cycle phases very closely. If one adopted the S_t coming from any dating program, such as BBQ, then it is unlikely that one could use S_t as a regressor, since it is a function of Δy_{t+1} and Δy_t . In such cases it would be necessary to use S_{t-2} as an instrument for S_t . Another case would be when S_t is included in a VAR with other variables z_t . Regressions of z_t against S_{t-1} (and S_t if it is an SVAR that is being estimated) would then be involved, and these would produce inconsistent estimators of the coefficients of S_t and S_{t-1} , unless an instrumental variable approach is taken.

In a good deal of the literature the S_t are taken to be independently distributed, whereas it is clear from the analysis of preceding sections that they are rarely that. Apart from the situation when a calculus rule is employed for dating yearly data which follows a pure random walk with no conditional heteroskedasticity, there will almost always be some serial correlation in them. The importance of recognizing this first came up when trying to test synchronization of cycles, where one looks at the correlation between S_{xt} and S_{yt} , where these might (say) represent the cycles of two different countries. Since the correlation between S_{yt} and S_{xt} is a function of the coefficient of the regression of S_{yt} on S_{xt} , testing if the former is zero involves testing whether the latter is zero. But the error term in such a regression would inherit the serial correlation properties of S_{yt} (as well as the fact that the S_{yt} is conditionally heteroskedastic), and so we need to make any t-statistics robust to that characteristic. As Harding and Pagan (2006) found, there could be extremely large differences in the t-statistics that were made robust to serial correlation and heteroskedasticity (HAC) compared to those that

weren't. Using HAC standard errors could dramatically change conclusions with respect to synchronization.

It is also possible that the equations of interest contain latent variables. An example would be the Qual-VAR model of Deuker (2005). In this z_t are observable variables and ψ_t will be a latent variable. A simplified form is below

$$z_{1t} = \alpha_{zz}z_{1t-1} + \alpha_{z\psi}\psi_{t-1} + \varepsilon_t \quad (16)$$

$$\psi_t = \alpha_{\psi z}z_{t-1} + \alpha_{\psi\psi}\psi_{t-1} + v_t, \quad (17)$$

where the shocks ε_t, v_t are normally and independently distributed with a zero expectation. Of course there are many other models like this that have latent variables e.g. a VAR driven by MS processes.

3.2 The State Dynamics Equations

To estimate the equations of interest mostly requires that one complete the system by augmenting them with equations describing how the states depend on variables in the equation of interest and how they evolve over time. One can distinguish two main approaches

3.2.1 Direct Mapping of States to Observable Variables

If one knew exactly how the states were constructed then this would be the standard approach to specifying state dynamics. As described earlier they would evolve as a Markov Chain with transition probabilities that can be derived from their recursive equations in (13).

3.2.2 Indirect Mapping of States to Observed Variables via Latent Variables

Often the states being used in the analysis are described as associated with the realization of some latent variable. Designating this as ξ_t it is frequently assumed that $S_t = 1(\xi_t > 0)$. Then an equation needs to be provided for ξ_t . In its simplest variant $\xi_t = x_t'\beta + e_t$, where x_t is observable and e_t is *n.i.d.*(0, 1). Consequently, $\Pr(S_t = 1) = \Phi(-x_t'\beta)$ and a Probit model is used to describe the evolution of S_t . Therefore any dynamics in x_t may carry over to S_t . There has however been a tendency to include S_{t-1} into x_t to produce

extra dynamics in S_t e.g. Candelon et al. (2010) recommend this for Early Warning System models while many working with published business cycle indicators as the S_t have followed Deuker (1997) in including S_{t-1} in x_t . Clearly there is a problem in doing this when using the NBER business cycle S_t , since previous analysis has shown that S_{t-1} is a function of outcomes in t and $t + 1$, and so cannot be treated as pre-determined, something required in the standard Probit approach.

In any case it is hard to see how S_{t-1} would logically appear in the single index of the Probit model. If states were censored, so that they followed a two period minimum phase as in business cycle applications), then the evolution would be governed by

$$S_t = S_{t-1}(1 - S_{t-2}) + S_{t-1}S_{t-2}[1 - (1\{\Delta y_t < 0, \Delta_2 y_{t+1} < 0\})] + (1 - S_{t-1})(1 - S_{t-2})[1\{\Delta y_t > 0, \Delta_2 y_{t+1} > 0\}], \quad (18)$$

and this would obviously not justify putting S_{t-1} *inside* Φ_t . Indeed, in this case we would be dealing with a double-index Probit model (at least) and not the single index one.

Two other approaches try to change the amount of dependence in S_t . One is Deuker's (2005) Qual-VAR where the latent process assumed to underlie the S_t is given some independent dynamics. Deuker estimated this by estimating (16) and (17) along with $S_t = 1(\psi_t > 0)$. In the descriptions of his model the variables in z_t seem to be the levels of variables such as GDP, but in his code they are growth rates. Hence, following his code, one might think that ψ_t is the growth in economic activity, and then one would rationalize this as a calculus rule applied to finding turning points in the level of economic activity. The problem with that is his use of NBER states, as we know that they do not use a calculus rule.

It would be more appropriate to adjoin the equations (16) and (17) with (13). These equations could then be estimated. The simplest approach would be to use an indirect estimation procedure in which a VAR in z_t and S_t is taken to be the auxiliary model. This approach preserves what is known about how the NBER states are found. A quick approximation to it is to set up a VAR composed of the observables with S_t replacing the latent variable. The logic of this is that, since $S_t = 1(\psi_t > 0)$, we know from Kedem (1980) that the autocovariance function of S_t would be a function of that for ψ_t . Hence this suggests that we can capture the effects of a latent variable (

to a linear approximation) by estimating the proposed observables-VAR. Of course it may be that a much higher order VAR is needed if only observables are used. But the method has the advantage of enabling one to exploit many of the features of VARs. In particular we could simply predict S_t using standard VAR software. Obviously it would be interesting to see how far away we would be when forecasting recessions using this observables-VAR than the latent variable Qual-VAR. For the 2001 U.S. recession the one period ahead probability is .32, from the proposed observables VAR, which is well below the probability that Deuker(2005) gave for the Qual-VAR, but the program used to compute that probability seems to be defective. If one simulates data from the program one finds that the unconditional probability of a recession implied by the Qual-VAR model is almost 50% higher than that in the data. This means that there was a bias towards forecasting recessions too often and it would show up in the one-period ahead probability of a recession. It seems that the program difficulties stem from how one treats unstable VARs in the simulations. Recently, Harding (2011) has tried to adjust the program for these difficulties.

Another way of inducing the extra dependence is to allow the Probit model coefficients β_t to be stochastic and to evolve as well. This is a recent proposal of Bellégo and Ferrara (2009). Again this would be expected to introduce some extra non-linear dependence and it would be interesting to effect a comparison of it with other methods of making a prediction.

4 Predicting Binary Recurrent States

4.1 One Period Ahead Recession Predictions

Using (13) and seeking to predict S_{t+1} we have

$$\begin{aligned}
 S_{t+1} &= S_t(1 - S_{t-1}) + S_t S_{t-1}(1 - \mathbf{1}(\Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0)) \\
 &\quad + (1 - S_t)(1 - S_{t-1})\mathbf{1}(\Delta y_{t+1} > 0, \Delta_2 y_{t+2} > 0)
 \end{aligned}
 \tag{19}$$

Mostly we are interested in predicting recession given the economy is in an expansion at time t and $t-1$. Such an assumption is not always true but it is useful to remove that source of uncertainty. In such a case $\{S_t = 1, S_{t-1} = 1\}$ and the prediction of S_{t+1} given information F_t will be

$$E(S_{t+1}|F_t) = E\{1 - \mathbf{1}(\Delta y_{t+1} \leq 0, \Delta_2 y_{t+2} \leq 0)|F_t\}.$$

To forecast recessions it is useful to transform to $R_t = 1 - S_t$, as then a recession is $R_t = 1$ and

$$\begin{aligned} E(R_{t+1}|F_t) &= E\{\mathbf{1}(\Delta y_{t+1} \leq 0, \Delta_2 y_{t+2} \leq 0|F_t)\} \\ &\leq E\{\mathbf{1}(\Delta y_{t+1} \leq 0)|F_t\} \\ &= \Pr(\Delta y_{t+1} \leq 0|F_t) \end{aligned}$$

Hence this sets an upper bound to the probability of predicting a recession. It draws attention to the need to predict negative quarterly growth one period into the future. Because this is a common element in virtually all definitions of a recession e.g. Fair (1993) and Anderson and Vahid (2001), it is useful to focus upon the size of this probability in the first instance. Indeed, it is a very quick way of getting an impression of why recessions will be difficult to predict from any model as it emphasises that one needs to be able to predict future growth rates in activity from past data. In the event that future growth depends mainly on unpredictable future shocks then one will find there is little predictive power from any model. An example of this happening occurs with the model of real/financial interactions in Gilchrist et al (2009). There the model predicts average expansion and contraction durations in per capita activity of 14.2 and 4.3 quarters respectively, which is a reasonable match to the durations in the data. But, if one suppresses the current (unpredictable) shocks in the model, it would produce durations of 30.8 and 3.7 quarters respectively i.e. current shocks are incredibly important to the cycle outcomes.⁵ Precisely defining a cycle in terms of a binary random variable is a key element in enabling one to grasp what should be focussed on when asking if recessions can be predicted.

4.2 Multi Period Ahead Recession Predictions

We want to look at forecasts of the states more than one step ahead when the economy is in an expansion at the prediction point i.e we know $S_t = 1$ and $S_{t-1} = 1$. E_t will designate an expectation taken with respect to an information set which includes S_t, S_{t-1} and the history of variables Δz_t . From the recursion (18) we would have

⁵To make this more concrete suppose that we had a series y_t that evolved as $y_t = \rho y_{t-1} + e_t$, where e_t is white noise. Then the exercise performed on Gilchrist et al (2009) effectively compares the cycle in y_t with that in $y'_t = y_t - e_t$.

$$E(S_{t+1}|F_t) = E[(1 - \wedge_t)|F_t]. \quad (20)$$

so that $E(R_{t+1}|F_t) = E(\wedge_t|F_t)$. A two period ahead forecast of S_{t+2} conditional on $S_t = 1, S_{t-1} = 1$ will be constructed from the recursion

$$\begin{aligned} S_{t+2} &= S_{t+1}(1 - \wedge_{t+1}) \\ &= (1 - \wedge_t)(1 - \wedge_{t+1}), \end{aligned} \quad (21)$$

as $E(S_{t+2}|F_t) = E((1 - \wedge_t)(1 - \wedge_{t+1})|F_t)$. This means

$$\begin{aligned} E(R_{t+2}|F_t) &= E(\wedge_t + \wedge_{t+1} - \wedge_t \wedge_{t+1} |F_t) \\ &= E(\wedge_t - \wedge_{t+1}(\wedge_t - 1)|F_t) \\ &\leq E(\wedge_t), \end{aligned}$$

and so the recession probability forecasts declines as we move from one period to two periods ahead.

Looking at forecasts three periods ahead the recursion gives

$$\begin{aligned} S_{t+3} &= S_{t+2}(1 - S_{t+1}) + S_{t+2}S_{t+1}(1 - \wedge_{t+2}) \\ &\quad + (1 - S_{t+2})(1 - S_{t+1}) \vee_{t+2} \\ &= S_{t+1}(1 - \wedge_{t+1})(1 - S_{t+1}) + S_{t+1}(1 - \wedge_{t+1})(1 - \wedge_{t+2}) \\ &\quad + (1 - S_{t+1}(1 - \wedge_{t+1}))(1 - S_{t+1}) \vee_{t+2} \\ &= (1 - \wedge_t)(1 - \wedge_{t+1}) \wedge_t + (1 - \wedge_t)(1 - \wedge_{t+1})(1 - \wedge_{t+2}) \\ &\quad + (1 - (1 - \wedge_t)(1 - \wedge_{t+1})) \wedge_t \vee_{t+2}, \end{aligned}$$

with the recession probability declining again. This suggests that in terms of assessing likely predictive success we should check the one period ahead predictions.

4.3 One Period Ahead Prediction of Turkish Recessions

4.3.1 Models using Information on Δy_t Only

As just foreshadowed we will focus upon the ability to predict a negative growth rate i.e. $\Pr(\Delta y_{t+1} < 0|F_t)$. Our data will be for Turkey although

elsewhere we have employed the same techniques for examining the Euro Area and US business cycles - Harding and Pagan (2010). It is clear that we need to have some model for Δy_t in order to evaluate predictive success (at least for understanding why a recession is being predicted). However, even if there is no formal model one could still look for correlations between future negative growth and some variables with the potential to predict this.

We begin by assuming that the model for Δy_t has the form $\Delta y_t = \phi_1 + \phi_2 \Delta y_{t-1} + e_t$, where e_t is $n.i.d(0, \sigma^2)$. A regression then gives $\Delta y_t = .003 + .64 \Delta y_{t-1} + .013 e_t$ and $\Pr(\Delta y_{t+1} < 0 | F_t) = \Phi(-[(.003 + .64 \Delta y_{t-1}) / .013])$. Using this model Table 1 gives the probability of predicting negative growth at the beginning of each of the six recessions identified in Pagan(2010).⁶ These recessions were 1988:4-1989:2, 1991:1-1991:2, 1994:2-1995:1, 1998:4-1999:4, 2000:1-2000:4 and 2008:4-2009:3. A second model involves regressing Δy_t against $\Delta y_{t-1}, S_{t-1}$ and S_{t-2} , and then using the predictions from this under the assumption that an expansion holds at the prediction point t i.e. $S_{t-1} = 1, S_{t-2} = 1$. Clearly the latter is information that can be exploited for forecasting, although it is unclear if either S_t or Δy_t would be known at the prediction point. In practice we rarely know what the growth rate in the current quarter is e.g. in Australia the best we would get would be GDP growth for $t - 1$ in quarter t . Even then this quantity can be subject to substantial revision and even a possible sign change. In terms of forecasting recessions this has two consequences. One is that it will no longer be the case that S_t can be known. If it was the case that S_{t-1} was known to be unity, then a positive Δy_t would mean that $S_t = 1$, since the peak in y_t would not be at $t - 1$. But if we don't know Δy_t then it might be negative. Since a negative growth can occur in an expansion, whether S_t is either 0 or 1 will not be known, and so we will need to predict this as well as $\Delta y_{t+j} (j = 1, 2)$. As Table 1 shows, when .5 is the threshold probability, neither model would have predicted the recessions that eventuated.

⁶Using $\Delta y_{t-1}, \Delta y_{t-2}$ and Δy_{t-3} as regressors does not change any conclusions.

Table 1 : Probabilities of Predicting Negative Turkish Growth One Quarter Ahead of Recession

Prediction At t /For $t + 1$	Model 1	Model 2
1988:3/1988:4	.29	.38
1990:4/1991:1	.11	.22
1994:1/1994:2	.20	.31
1998:3/1998:4	.29	.38
1999:4/2000:1	.11	.22
2008:3/2008:4	.35	.43

To put these numbers into context, since 24% of the time was spent in recession, if you just allocated a value of .24 every period you would be doing better than trying to exploit the information available in growth rates. A similar result holds for the Euro area and the US, with the probabilities for the latter varying between .06 and .27 for the recessions since 1953. It should be noted that the unconditional probability of a recession over the period 1953/2 to 2009/3 in the US is .16.

It is also useful to look at the probabilities from Model 1 as the 2000/2001 recession unfolded (clearly since Model 2 uses information on S_t in the recession it is not feasible to use that model)

Table 2 :Probabilities for Negative Growth, 2000 Turkish Recession

	Model 1	R
2000:1	.11	1
2000:2	.46	1
2000:3	.85	1
2000:4	.77	1
2001:1	.86	0
2001:2	.31	0

This is a typical pattern - the first period of the recession is predicted with very low probability but then rises as the recession gets underway.

4.3.2 Can Non-linear Models of GDP Growth Help?

In the previous sub-section it was found that a linear function of the past history of Δy_t was not particularly useful in predicting Turkish recessions. One might seek to make the process for Δy_t a non-linear function of past growth. In fact the inclusion of S_{t-j} in the Model 2 regression of the previous sub-section essentially does this, although because of the nature of S_t it

produces a linear single index within the Probit function. Instead one might wish to make the index a non-linear function of past growth. A recent MS model fitted to Turkish data is Senyuz et al (2010).⁷ Their MS model is quite complex, with two states, an AR(3) structure with constant parameters, and variances that are state dependent. Fitting their model to the current data produces quite a lot of periods in which growth becomes negative - twice what it is in the current data set. A number of other MS models were fitted but they all failed simple tests like this. Moreover, they often had rather odd descriptions of the states e.g. an MS with an AR(2) structure and a constant variance suggested that the first state had a mean growth of .19% and an expected duration of 14 quarters, while the other state had values of 3.5% and 3 quarters, which is the converse of what one would expect. Similar problems with MS models for the US and the Euro Area were also identified in Harding and Pagan (2010). As explained before it seems likely that it may reflect convergence problems in getting estimates of the parameters due to the labelling identification issue.

4.3.3 Using Multivariate Information - A VAR Model

One might have some information available other than the history of the variable that ultimately determines the indicators S_t e.g. it has been suggested by many authors that the spread between short and long rates of interest can be useful in predicting recessions - see Estrella and Mishkin (1998). Alternatively, this information may be put into a model such as a VAR, or even a DSGE model, and the estimated model would then be used to provide a forecast. The model might even have a latent variable in it e.g. as in the Qual-VAR. In the latter case, after estimation we will be able to simulate such a model and then construct simulated values on the variables that determine S_t in the prediction period. These can then be used to predict a recession. This is quite an old idea - Fair (1993) - but mostly the criteria used for defining recessions etc. in such experiments have been non-standard e.g. predicting whether there would be two periods of negative growth.

To look at the role of extra information in predicting recessions a small structural Vector Autoregression (SVAR) model was fitted to Turkish data from 1990:3 until 2010:1.⁸ The length of sample was determined by the avail-

⁷It should be noted that the data used by Senyuz et al (2010) is not seasonally adjusted and the data I am using has been adjusted.

⁸Sometimes the sample started at 1990:4 and ended at 2009:4, depending on the lags

ability of a short run interest rate (i_t). The variables fitted were the logs of exports (x_t), GDP (y_t), Gross National Expenditure (n_t) - "absorption" in international economic models - CPI inflation (π_t) and the real exchange rate (q_t).⁹ The model is a smaller version of that used by Dungey and Pagan (2000) for Australia, and has close connections with that used in Catão and Pagan (2010) when modelling Brazil and Chile. In the latter paper a model based on a typical New Keynesian model for an open economy was augmented with extra variables if the data supported such additions. Here we do not have the forward looking expectations in equations that appeared in Catão and Pagan (2010). For our purpose this did not seem necessary as the expectations are always replaced with observable variables and so would show up as extra regressors if required. Essentially the equations can then be solved to determine a data generating process for Δy_t .

A few comments on the SVAR equations in (22)-(28) are in order. First, variables with a tilde are deviations from a fitted deterministic trend and so can be regarded as "gaps". The trends are much the same for GDP and GNE but that for exports is almost twice as large. Exports typically grow faster than GDP for many countries and this is handled in the trade literature using gravity models. Some of this disparate behaviour comes about due to the removal of trade barriers. As these are largely exogenous to the economic outcomes of the country being examined, we simply allow the trend growth in exports to be higher than GDP. A second order SVAR was taken to be the reference point, reflecting the fact that many New Keynesian models imply a VAR(2) as their solved solution. The data strongly supports this for some equations. If the second lags of variables were not significant they were deleted.

Because the model is recursive OLS was applied to estimate the coefficients. Equations (22)-(28) provide the estimated coefficients with the absolute values of the t-ratios below the coefficients. The ε_t^j have standard deviations of unity, and so the scalar multiplying them is the standard deviation of the shock. The shocks were generally uncorrelated, the exception

and data availability.

⁹Both exports and GNE seemed to have a seasonal pattern, and thus these series were smoothed by a fourth order moving average, just as for GDP. After this seasonal adjustment, the exports, GNE and GDP data were converted to percent deviations. Data for interest rates and inflation have been converted to annual percentages and the log of the real exchange rate was multiplied by 400 to be consistent with these units.

being those associated with the real exchange rate and GNE equations.¹⁰

$$\tilde{x}_t = \underset{(15.8)}{1.55}\tilde{x}_{t-1} - \underset{(5.5)}{0.56}\tilde{x}_{t-2} - \underset{(2.2)}{0.007}q_{t-1} + 1.86\varepsilon_t^x \quad (22)$$

$$\tilde{n}_t = \underset{(21.1)}{1.67}\tilde{n}_{t-1} - \underset{(9.5)}{0.78}\tilde{n}_{t-2} - \underset{(0.9)}{0.013}rr_{t-1} + 1.98\varepsilon_t^n \quad (23)$$

$$\begin{aligned} \tilde{y}_t = & \underset{(11.9)}{0.86}\tilde{y}_{t-1} - \underset{(1.6)}{0.07}\tilde{y}_{t-2} + \underset{(22.0)}{0.53}\tilde{n}_t - \underset{(10)}{0.41}\tilde{n}_{t-1} - \underset{(1.5)}{0.001}q_{t-1} \\ & + \underset{(3.7)}{0.022}\tilde{x}_t + 0.46\varepsilon_t^y \end{aligned} \quad (24)$$

$$\pi_t = \underset{(3.5)}{1.33}\tilde{y}_t - \underset{(11.3)}{0.27}q_{t-1} + 16.4\varepsilon_t^\pi \quad (25)$$

$$i_t = \underset{(12.4)}{0.77}i_{t-1} + \underset{(2.7)}{0.14}\pi_t + \underset{(0.6)}{0.09}\tilde{y}_t - \underset{(1.5)}{0.03}q_{t-1} + 7.1\varepsilon_t^i \quad (26)$$

$$q_t = \underset{(12.3)}{0.81}q_{t-1} + \underset{(2.4)}{0.44}(rr_t - rr_t^*) - \underset{(2.4)}{0.46}\pi_{t-1} + 27.7\varepsilon_t^q \quad (27)$$

$$rr_t = i_t - \pi_t \quad (28)$$

Given the estimated parameters of the SVAR above we find that

$$\begin{aligned} \Delta y_{t+1} &= .85 + .034\tilde{x}_t - .14\tilde{y}_t + .475\tilde{n}_t - .0069rr_t - .0012q_t \\ &\quad - .012\tilde{x}_{t-1} - .07\tilde{y}_{t-1} - .413\tilde{n}_{t-1} + u_{t+1} \\ &= \psi_t + u_{t+1} \\ u_{t+1} &= .022\varepsilon_{t+1}^x + .53\varepsilon_{t+1}^n + \varepsilon_{t+1}^y \end{aligned}$$

It is also possible to find a (much lengthier) expression for Δy_{t+2} in terms of information available at t . The first period probability of getting a negative growth rate with the model is then computed using $\Pr(u_{t+1} < -\psi_t)$ under the assumption that the shocks are normal, and it will be $\Phi(-\sigma_u^{-1}\psi_t)$. By constructing an expression for Δy_{t+2} it is also possible to compute the probability of a recession at $t+1$ (given $S_t = 1, S_{t-1} = 1$) from the joint probability $\Pr(\Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0)$. We might expect that the model will have some success in predicting recessions. Using the test described previously, wherein the current unpredictable shocks are removed from output i.e. we construct the level of output y'_t by cumulating $\Delta y_t - .022\varepsilon_t^x + .53\varepsilon_t^n + \varepsilon_t^y$, we find that the durations of expansion and contractions are 10.9/4.9 for y_t and 13.4/4.9

¹⁰Re-estimation with Dynare imposing uncorrelated shocks produced only small changes to the coefficients.

for y'_t . Thus there is clearly some potential for model predictability of the cycle indicators.

Table 3 compares the probability of negative growth from the earlier Model 1 and the VAR model above. Also present is the probability that there will be a recession from the VAR model. Of course, as shown earlier, the latter must be smaller than the former.

Table 3 : One Quarter Ahead Probabilities of Predicting Turkish Negative Growth and Recession

PredAt t /For $t + 1$	Model 1	Pr(neg growth), VAR	Pr($R_t = 1$), VAR
1990:4/1991:1	.11	.23	.16
1994:1/1994:2	.20	.33	.28
1998:3/1998:4	.29	.57	.48
1999:4/2000:1	.11	.04	.03
2008:3/2008:4	.35	.81	.78

There are some items of interest in the table. First, the recession probability and the probability of negative growth are not too far apart and this points to the utility of looking at the latter when seeking a quick assessment of whether any model will be useful in predicting recessions. Indeed, we have found this to be true more generally and it is therefore always useful to ask what a new suggested model for Δy_t brings to the task of predicting negative growth (rather than growth generally as it might be capable of performing well on the large majority of positive growth rates but failing dismally on the negative ones). Second, the model has dramatically improved the prediction of two of the recessions and (with the exception of 2000) improved on the univariate information. Lastly, the performance in predicting the recent recession is interesting. In 2007/2 the probability of a recession was just .15 but it then rose as exports declined rapidly relative to what their historic growth rate was (this resulting in a larger export gap). By 2008:2 the model was predicting a recession in the next period of .65. It is noticeable that Senyuz et al (2009) set 2008:3 as the beginning of the recession.

5 Conclusion

There has been increasing interest in constructing and using S_t in policy and historical analysis. We are now seeing S_t being used in regressions, VARs, panel data etc. To use these properly we need to understand the nature of S_t , as this determines exactly how we need to (modify) these estimation

methods. Prediction of S_t needs to be carefully done by exploiting its nature. The lecture attempted to provide a framework for analysing the issues and pointing out some solutions.

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