Patterns and Their Uses*

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Abstract

Three major themes have emerged in the literature on patterns. These involve pattern recognition, pattern matching (do a set of observations match a particular pattern?) and pattern formation (how does a pattern emerge?). The talk takes up each of these themes, presenting some economic examples of where a pattern has been of interest, how it has been measured (section 2), some issues in checking whether a given pattern holds (section 3), what theories might account for a particular pattern (section 4), and the predictability of patterns (section 5). Most attention is paid to judging macroeconomic models based on their ability to generate macroeconomic and financial patterns, and some simple tests are suggested to do this. Because sentiment and the origins of patterns are so inextricably linked in macroeconomics and finance we will spend some time looking at the literature which deals with the interaction of series representing sentiment with those representing macroeconomic and financial outcomes.

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1 Definition

The Wikipedia entry relating to a pattern defines it as a set of elements describing recurring events that repeat in a predictable manner. It elaborates on this by saying that if the elements have enough in common for the underlying pattern to be inferred then patterns can be based on a template or model. It notes that there are many types of patterns with different aspects - Penrose tiling (symmetry), fractal (chaotic), harmonic oscillation etc.

Three major themes have emerged in the literature on patterns. These involve pattern recognition, pattern matching (do a set of observations match a particular pattern?) and pattern formation (how does a pattern emerge?). The paper takes up each of these themes with some economic examples of where a pattern has been of interest and how it has been measured (section 2), some issues in checking whether a given pattern holds (section 3),
what theories might account for a particular pattern (section 4), and the predictability of patterns (section 5). Most of the material comes from work I have done in the past decade with a variety of co-authors, in particular Don Harding and Tim Robinson. I use the examples from this work since I understand the material better than other work that I reference but there is no doubt that the research I report on is just a small part of what is now a rapidly growing field of enquiry.

It might be said that many of the classic theories in macroeconomics were stimulated by patterns that observers felt were in the data e.g. Austrian economics was influenced by the idea that booms led to busts through over-expansion of credit and investment, since one could observe in history that large increases in credit (debt) led to depressions through severe contractions in investment owing to over-optimistic assessment of the return to capital. In the same vein Pigou’s theory of business cycles involved a somewhat similar mechanism but emphasized over-optimistic expectations being eventually replaced by pessimism and thereby initiating a substantial recession. In technical analysis patterns are taken to signify the beginning and end of bull and bear markets, and the pattern is generally isolated through an emphasis on the volume of trade, which is often motivated by references to sentiment. Keynes of course viewed sentiment as the main driver of investment decisions (although he waxed and waned on this) due to the difficulty of doing forecasts, and this has led to a substantial literature regarding ambiguity and its resolution e.g. Ilut and Schneider (2012). Because sentiment and the origins of patterns are so inextricably linked in macroeconomics and finance we will spend some time looking for patterns in those series, research which seems now to be attracting a good deal of attention in the wake of the GFC.¹

2 Pattern Recognition

Although initially recognition of patterns largely stemmed from inspection of graphs e.g. booms and busts in economies and asset prices could be seen this way, and even now in areas like technical analysis this is still probably the major way patterns such as "head and shoulders" are determined, there has been an increasing emphasis upon automated methods for detecting patterns. Such a development probably came because the findings of these methods can

¹There is of course a literature on patterns in microeconomics as well.
be replicated and utilized for many purposes. Moreover, they will lead to a summary of the pattern by a set of pattern descriptors (or variables), often of a binary nature. We will distinguish two main automated methods that have emerged. These emphasize either turning points in a series (or series) or clustering of some feature in a series (or series). In each of these we find that the pattern is derived from the data either by some simple rule or by utilizing some model to generate rules. Models tend to produce complex rules and they may be hard to identify and to replicate. Nevertheless, because many researchers have been trained in the fitting of models, it is perhaps inevitable that these have become popular. I am rather sceptical of their advantages and will make some reference to this later in the paper.

2.1 Through Turning Points

2.1.1 Simple Rule Based Methods

Because turning points are invariant to monotonic transformations those in a series \( Y_t \) will be the same as those in \( y_t = \log(Y_t) \). Then, following calculus, one might identify turning points at \( t \), such as a local peak, by requiring that \( \{y_{t-1} < y_t > y_{t+1}\} \) i.e. \( \{\Delta y_t > 0, \Delta y_{t+1} < 0\} \). All turning point rules are variants of this idea. When series are volatile one either smooths the data before applying the rule or one extends the criterion to have a bigger window e.g. a local peak happens if \( \{y_{t-k}, ..., y_{t-1} < y_t > y_{t+1}, ..., y_{t+k}\} \). This is the type of rule that is used in the NBER determination of business cycle turning points - with monthly data one sets \( k = 6 \). Of course one might use both smoothing and a relatively small window. In the event that series are smoothed e.g. by first forming \( z_t = \sum_{j=0}^{M} \omega_j y_{t-j} \), and then turning points in \( z_t \) are found with \( k = 1 \) it is clear that a local peak will be occur when.\(^3\)

\[
\Delta z_t = \sum_{j=0}^{M} \omega_j \Delta y_{t-j} > 0, \Delta z_{t+1} = \sum_{j=0}^{M} \omega_j \Delta y_{t-j+1} < 0 \tag{1}
\]

But this is rarely the end of the process. If \( k > 1 \) an application might produce a case where one ends up with two peaks in a row, whereas we would

\(^2\)Local troughs of course will involve \( \{y_{t-1} > y_t < y_{t+1}\} \). Because the rule has the same structure as that for local peaks we will concentrate upon the latter in what follows.

\(^3\)One might use a two sided moving average rather then the one-sided one described here.
want peaks and troughs to alternate. So we would need to eliminate one of these, presumably the one with the smallest $y_t$. Even after this is done we might feel that either the time between peaks and troughs (phases) or peaks and peaks is too small i.e. downturns do not last for long enough or that a complete cycle is too short, so we might wish to impose some minimum durations. This in turn will lead to the elimination of some peaks and troughs and we will need to make sure that the resulting set alternate. Thus in NBER dating work upturns and downturns must last six months while a peak to peak movement must be 15 months. Programs that enforce these rules can be quite complex. Sometimes they work well with actual data but, when faced with simulated data, they prove less satisfactory. The BBQ program, which simplified the algorithm in Bry and Boschan (1971) in the way set out in Harding and Pagan (2002), required some years of refinement, and it was only with James Engle’s modification (MBBQ) that it has proved reliable.⁴

Once turning points have been found one can construct a binary variable $S_t$ that is unity when an expansion holds at $t$ and zero when it is a contraction. When $k = 1$ this could be written as $S_t = 1(\Delta y_t)$. The $S_t$ can then be used to recover the periods of time in which expansion and contraction phases hold and so it is a pattern descriptor. Of course there are other rules that might be used to define $S_t$ e.g. Anderson and Vahid (2001) for recessions. Other modifications might involve extra constraints that one thinks it desirable to apply e.g. quantitative ones which eliminate shallow downturns by applying the criterion that a downturn occurs only if $\Delta y_{t+1} < -\alpha (\alpha > 0)$, rather than $\Delta y_{t+1} < 0$. This happens when defining "sudden stop" events.

These construct a binary $S_t$ with the sudden stop happening when the annual decline in capital inflows ($y_t$) is either more than two standard deviations away from the mean annual growth rate or it shows a decline exceeding a predetermined fraction of GDP. Other examples are work that looks at subsets of expansions and contractions. One of these would be to define $S_t = 1$ only if the upturn in $y_t$ exceeds its previous peak e.g. Claessens et al (2012) used this, saying it describes a "recovery" phase. Another strand of research defines booms and busts as extremely large movements in expansions and contractions. Thus Bordo et al. (2009) define booms as bull markets that either have a duration of 3 years and an annual rise in the real stock price of 10% or which last 2 years with an annual rise of 20% while Busts last at least 12 months and feature an annual decline of at least 20%. Of course in

⁴It is available at http://www.ncer.edu.au/data/ in Gauss, Matlab and Excel forms.
this case one would have a three state situation - booms, busts and normal times- and we would need two binary variables to capture these outcomes. Finally Frankel and Rose (1996) define a currency crisis as a nominal depreciation of the currency of at least 30 percent that is also at least a 10 percent increase in the rate of depreciation compared to the year before while Luc and Valencia (2008) define systemic banking crises as occurring when there are sharp increases in default rates accompanied by large losses of capital (although they also use judgement to define the binary variable).

Cycles have been investigated in a great many series. The business cycle is the classic example but there are cycles in asset prices such as equity and house prices, interest rate cycles, credit cycles, commodity prices, IPO cycles e.g. Claessens et al (2011), Cashin et al (2002), Ibbotson and Jaffe (1975). Basically all these series have recurrent upturns and downturns that are called cycles and they are summarized by the pattern variable $S_t$. Different names are often given to the phases- bull/bear, hot/cold, boom/bust etc.

Technical analysis often works with turning points, as seen from the graph below of a "heads and shoulders" pattern. Let P be a peak and T a trough. Then the head and shoulders formation is clearly the pattern \{P,T,P,T,P\}. Of course such a pattern will occur in any series after we have made sure that peaks and troughs alternate, so by definition there must always be a head and shoulders pattern. To make it meaningful an extra layer of censoring is imposed through some conditions viz. that the volume of trade must be high in the expansion leading to the left shoulder and it is followed by a slide to a low volume, after which there is a rise to a very heavy trading volume during the run up to the head. Volume thereafter declines and there is less volume in the right shoulder than the left. Lo et al (2000) set up a number of technical analysis rules through turning points. They actually locate turning points in a smoothed series rather than in the raw data.

2.1.2 Model Based Methods

Perhaps the most important of the models used to find turning points are MS models introduced into econometrics by Hamilton (1989). In their simplest form these are

$$\Delta y_t = \mu_0(1 - \xi_t) + \mu_1 \xi_t + \varepsilon_t$$
Figure 1: Head and Shoulders Pattern
where $\xi_t$ is a first order Markov Process. The model is fitted and estimates of $t = \Pr(\xi_t = 1 | \Delta y_t, ..., \Delta y_0)$ are found\(^5\). One can’t find an analytical expression for the probability but Harding and Pagan(2003) argued that it could be quite closely approximated (at least for US GDP data) as $\delta + \sum_{j=0}^{M} \omega'_j \Delta y_{t-j}$, where $\omega'$ and $\delta$ came from the Kalman filter. Of course this does not establish turning points. One needs to add on the constraint that an upturn (downturn) holds at $t$ if the probability exceeds (is less than) some critical value $c$ i.e. if $\phi_t > c$ ($< c$). Consequently, a peak will obtain at $t$ if $(\phi_t > c, \phi_{t+1} < c)$ i.e.

$$\{ \delta + \sum_{j=0}^{M} \omega'_j \Delta y_{t-j} > c, \delta + \sum_{j=0}^{M} \omega'_j \Delta y_{t-j+1} < c \},$$

and this produces a binary pattern variable $\xi_t$. Comparing (1) and (2) one sees that the rules can be thought of as similar - the MS model effectively applies a window width $k = 1$ to smoothed data, where the smoothing is determined by the MS model. So in some sense the smoothing adapts to the data. Even if the smoothing weights $\omega'_j$ were the same as for the simple rules ($\omega_j$), unless $c - \delta = 0$ we would find different answers. Of course the simple rule is often applied to data with values of $k$ equal to two (for quarterly data), but this can be mimicked by the MS model if a two-sided rule is used to find $\phi_t$. With that set-up the weights $\omega'_j$ may die away quite quickly, and so only $\Delta y_{t-1}, \Delta y_t, \Delta y_{t+1}, \Delta y_{t+2}$ are effectively present in the summation.

In Hamilton’s original paper $c = .5$, but many other values have been proposed. Chauvet and Morais (2010) for example set $c$ equal to the sample mean of $\Pr(\xi_t = 1 | F_t)$ plus one standard deviation. Candelon et al. (2010) consider a range of methods to determine an optimal $c$ which are various functions of the Type 1 and Type 2 errors which would occur for any given value of $c$. Of these the most common method is to choose $c$ to minimize the ratio of false to true positive outcomes. Clearly such a criterion relies upon the fact that a "true" set of indicators have already been established. Often the fitted model and $c$ are varied until they produce a good match to the turning points found from simple rule based methods, which raises the question of why one wants to do the MS based models in the first place?

When applied outside the US context the simple MS model has been found to be inadequate. This has led to increasingly complex MS models.

\(^5\)Sometimes $\Pr(\xi_t = 1 | \Delta y_T, ... \Delta y_0)$ is used and this leads to a two-sided filter.
Thus the AR coefficients $\beta$ and $\sigma$ have been allowed to shift according to the latent state outcomes, while the transition probabilities have been made to depend upon predetermined variables and the duration of the phases (Filardo (1994)). More recently the latter have been allowed to be stochastic e.g. by Billio and Casarin (2010).

One issue with MS models is that there can be computational problems in finding an optimum to the likelihood (or computing the posterior). As Smith and Summers (2004, p2) say "These models are globally unidentified, since a re-labelling of the unobserved states and state dependent parameters results in an unchanged likelihood function". Applying MCMC type methods doesn’t resolve it as the labelling problem means that one is drawing from different densities and this can affect convergence of the sampler. The labelling issue has been discussed a good deal in statistics and a number of proposals have been made to deal with it e.g. Frühwirth-Schnatter (2001) but few of these seem to have been applied to empirical work with MS models in economics.

Although the MS model has been the most widely used vehicle for establishing model based turning points there are some other candidates. A recent one involves the same idea as an MS model in that there is a binary signal distorted by noise, but now it is proposed that the binary signal should be extracted without specifying a parametric model. This method has been termed "non-parametric decoding" by Fushing et al (2010). It is hard to know what to make of it. Table 1 of their paper shows that when applied to US data it has two more recessions than the NBER distinguishes, despite using the same information as the NBER Dating Committee. As well, for the U.K. it has five recessions in the 2000s, whereas GDP movements would only point to two. Indeed, in the U.K. there was no negative growth in GDP until 2008/2, so it is hard to see where five recessions come from. Of course one can argue that GDP should not be the only series used in dating recessions, although Burns and Mitchell ( p 72) did say that "Aggregate (economic) activity can be given a definite meaning and made conceptually measurable by identifying it with gross national product". Because the method seems technically very complex it is hard to know why it seems to produce "false" recessions.\(^6\)

\(^6\)There is a similar problem for Australia, which supposedly had recessions in 1994/5, 1996 and 2000/2001, even though policy makers clearly did not notice these.
2.2 Through clusters

Often a pattern is inferred by seeing "clustering" activity. Perhaps the best known example in finance would be volatility clustering in a single series i.e. periods of high volatility occur together and the state has some persistence. However, it may also be that clustering pertains to multiple series in which a given feature occurs at much the same point in time e.g. downturns happen at much the same time in many industry output series or in GDP for different countries. In these latter cases identifying a cluster is essentially a question of compression. The classic example of such compression is the decision by the NBER Dating Committee who take turning points in a number of series representing economic activity and then find a single set of turning points around which the turning points of the individual series cluster.

It is often said that the patterns we see in macroeconomic and financial data strongly reflect movements in sentiment. Probably these ideas go back to the manic depressive theory of the business cycle revealed in the following remark about the 1820's recession in New South Wales by John Dunmore Lang (1852) (quoted in Goodwin (1966, p.220))

“in short the body politic of the colony has passed through a crisis of violent and unnatural excitement, which, according to the well-known maxim of Hippocrates, the father of medicine must necessarily be followed by a corresponding unnatural depression”.

Keynes of course was very much attracted to the idea that "animal spirits" were the key to decisions in the face of uncertainty, particularly those relating to investment, while Pigou thought that over-optimism was the cause of a boom and a subsequent recession when the optimistic expectations prove unfounded - Beaudry and Portier (2006). This suggests that we should pay attention to sentiment indicators to see what patterns emerge from them. In particular, as interest has turned strongly towards global cycles, we might note that if sentiment is really a key to cycles, then we should see clusters in the various series within a country and between countries.

2.2.1 Simple Data Based Rules

The idea behind this strategy is to say that the turning points in aggregate economic activity occur at those points in time that the turning points of the individual series cluster around. The problem then is to decide on the central tendency of these specific cycle turning points as this will be used as
the reference cycle turning point. How one proceeds depends on how many series one utilizes and the information available about where the turning point might be. The turning points can then be aggregated to obtain a reference cycle using the algorithm described in Harding and Pagan (2006) which essentially defines what is meant by a clustering of turning points. Consider a point in time \( t \) as a candidate for a turning point in aggregate economic activity. The turning points of the \( j^{th} \) series will lie at (say) \( t_j \) and we then measure the absolute distance between \( t_j \) and \( t \) calling it \( \tau_j = |t_j - t| \). Let the median of \( \tau_j \) be \( \phi(t) \). Then \( t^* \) would be taken to be the time when a reference cycle turning point occurs if \( \phi(t^*) \) has a local minimum value. In practice one needs to break ties e.g. if \( t^* \) and \( t^{**} \) both have a local minimum in \( f(t) \).

The method was based on the NBER procedures as documented by the late Geoffrey Moore and it was formalized by studying the spreadsheets used by Boehm and Moore (1984) when obtaining an NBER-like reference cycle for Australia. The algorithm seemed to be able to produce quite a good replication of the reference cycle published by the NBER and also the equivalent set of information for Australia. Variants of the algorithm have been used by Borriro (2012) for dating an aggregate credit cycle from three credit measures, and Canova and Schlaeper (2013) for constructing a Mediterranean reference cycle and Chauvet and Piger (2008) for real time dating of cycles.

Stock and Watson (2012) considered the case where there are many series available and one is looking to establish where a reference cycle turning point should be placed in an interval (called by them an episode) given that we know there is a turning point in that interval. Given an episode one can find the fraction of times among the series that a turning point is signalled at each period of the interval. Then one could choose the reference cycle turning point as being at \( t^* \) if the fraction of series that turn at that date is highest. This is equivalent to selecting the mode of the distribution of the specific cycle turning points, but Stock and Watson allow for the mean or median as alternative choices. In their application 270 series were used. Intervals for where a peak occurs were found using a "heatmap" showing the growth rates in the individual series. Very strong negative growth was coloured red and "the beginning of each red band suggests a reference cycle peak and the end of the band suggests a trough" (p 15). Heatmaps are certainly visually very striking and appealing. So in this case the broad pattern is coded through a colour scheme, although the exact details require an algorithm.
2.2.2 Model Based Rules

Although models have been used for formalizing the notion of clusters in cycles it seems more interesting to focus upon what seems to me to be an undeveloped theme viz. the relation between sentiment measures at a global level. One could look at turning points in sentiment measures but one is rarely interested in that pattern. Rather it is the coincide of high and levels of confidence that attract more attention. Without a definition of high and low it is hard to define a criterion to measure clusters. One possibility is to fit MS models to the series representing sentiment but this rarely gave satisfactory results, in the sense that one could isolate high and low values for the series. We examined sentiment measure for various countries in Europe, the US and Australia. Based on statistical tests we can say that they are stationary AR(2) processes (all the European ones and the Westpac/Melbourne Institute measure from Australia have this feature but not the Michigan Index of Consumer Sentiment from the US, the latter being an AR(1)). Often the indices exhibit heteroskedasticity of the EGARCH variety i.e. there is clustering of volatility.

Now a model often used in deciding on whether there is a cluster involves determining of there is a common factor among them, and one way to assess this is to look for principal components. Analysis of the series shows that the first component accounts for 64% of the total variance, the second component accounts for an extra 17% and the third another 8.5%. The weights on the first three components are given in Table 1.

As the first factor weights show there does seem to be very strong clustering of sentiment within the European countries, and even the US gets a reasonable weight in it. As the second factor reveals Australia is generally divorced from the European outcomes and is more related to the US. Clearly it would be interesting to continue this line of work with indices coming from Asian economies.

3 Pattern matching

3.1 Using the patterns to create "stylized facts"

An example often cited is the view expressed in Blinder and Maccini (1991) that a very large part of the reduction in GDP during recessions is associated with reductions in inventory investment. To assess the implications of their
Table 1: **Loadings for the First Three Principal Components**

<table>
<thead>
<tr>
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<th>PC1</th>
<th>PC2</th>
<th>PC3</th>
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<td>.25</td>
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<td>Belgium</td>
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<td>-.10</td>
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<td>Portugal</td>
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<td>-.37</td>
</tr>
<tr>
<td>Australia</td>
<td>-.03</td>
<td>.66</td>
<td>.54</td>
</tr>
</tbody>
</table>

observation it is useful to quantify it with the ratio $BM = \frac{\sum_{t=1}^{T}(1-S_t)\Delta^2K_t}{\sum_{t=1}^{T}(1-S_t)\Delta Q_t}$, where $S_t = 1$ if GDP ($Q_t$) is in an expansion phase and $S_t = 0$ if it is in a contraction phase. Hence this statistic records the average fraction of GDP declines across a recession that is associated with declines in inventory investment. It equals .77 for the 1959-2005 period (.78 over 1959-1983). This statistic is often used to support the view that inventory investment ($\Delta K_t$) movements seem to "account" for a large part of output decline.

In Maccini and Pagan (2013) we constructed a model that features inventories in the goods producing sector. A range of shocks are present in the model, including sales, technology and inventory cost shocks. After fitting the model to data it was simulated and business cycle features were constructed with the BBQ algorithm and then used to compute the BM statistic. The model produced a mean of BM of .88 (found by averaging over 500 sets of simulated data with 98 observations in each set). The range of variation that comes out of the simulations is quite large, meaning that the predictions are consistent with the data (BM=.77) at an extremely low level of significance. Now testing of the model showed that it captured many of the features of inventory movements and their relation to the business cycle quite well. Yet, if one eliminated inventories from the model by making them too expensive to hold, the business cycle scarcely changed. So the presence
of inventories was not responsible for the business cycle, yet 88% of the contraction in GDP across a recession showed up as inventory movements. It is clear from this that it is the fact that $K_t$ and $Q_t$ are endogenous variables, and there is a common factor among them that causes some coherence, which results in the feature measured by the BM statistic, but there is no causal connection to the business cycle.

### 3.2 Assessing Whether Impulse Responses Vary with a Pattern

There is an increasing tendency to use indicators of crises and cycles as variables in VARs to determine if impulse responses vary with them. But unless care is used this can lead to quite invalid results, since the cycle indicators $S_t$ are functions of shocks at $t + j$ ($j \geq 0$) i.e. to determine a peak at $t$ (using BBQ) one needs data on $\Delta y_{t+1}, \Delta y_{t+2}$. Hence, if one uses these as regressors, biases in parameter estimates will result, since the regressor is correlated with the error. To demonstrate this assume that the DGP for the quarterly variable $y_t$ whose cycle is to be measured and then used in a regression is

$$
\Delta y_t = .0055 + .33\Delta y_{t-1} + .0093\varepsilon_t, \quad \varepsilon_t \sim N(0,1).
$$

In this set-up impulse responses are invariant to the state of the cycle. Now suppose we measure the cycle in $y_t$ with the binary variable $S_t$ constructed with the BBQ algorithm. Following what has been done in some research studies we run a regression in which dynamics are postulated to depend upon $S_t$ i.e. we fit

$$
\Delta y_t = .0053 - .09\Delta y_{t-1} + .015 S_t \Delta y_{t-1} + .009 \varepsilon_t.
$$

So the impulse response function from this regression when $S_t = 1$ (expansion) is $(.06)^j$ while, when $S_t = 0$, it is $(.09)^j$. In reality there is no dependence of the impulse responses upon either expansions or recessions. Despite the fact that the addition of the interactive term $S_t \Delta y_{t-1}$ is highly significant it results in major distortions to the magnitudes, signs and persistence of impulse responses owing to the correlation of $S_t$ with $\varepsilon_t$. 
3.3 Using Patterns to Judge Econometric Models

Iacoviello (2005) developed a model in which there were two groups of households, differing by their discount factors. Patient consumers invested in capital and housing as well as possibly lending to the impatient households. The impatient households invested only in housing and were subject to a borrowing constraint, with the amount that can be borrowed being subject to a collateral requirement i.e. there is a fixed ratio of the value of the loan to the value of capital (the loan to value ratio). The borrowing constraint was assumed to always bind with equality so that the model may be log-linearized. The collateral is a durable good used in production - in the model it is referred to as housing.

A limitation of the Iacoviello (2005) model is that the quantity of housing in the economy is fixed. The lack of any supply response potentially amplifies the response of house prices to shocks and may therefore increase the impact of the borrowing constraint. Iacoviello and Neri (2010) (IN) introduce a similar second production sector into the Iacoviello model. The housing sector uses labour from both type of consumers, together with land and intermediate goods from the other production sector. Land is in fixed supply, but used to produce new housing, which may be purchased by either type of consumer. Technology in each production sector grows deterministically at different rates. The model also possesses sticky wages for each type of household in each sector, which considerably alters the dynamic responses of the model to shocks. There are also sticky prices in the retail sector. The decisions made by households and firms are much the same as those in models such as Smets and Wouters (2007), except that credit constraints can limit expenditures, and so changes in the value of collateral can potentially have effects on cycles.

The question we look at is whether the model can re-produce observed business cycle outcomes seen in the data that IN use? The business cycle dating program used was BBQ. Using the parameter values provided in Iacoviello and Neri (2010) we simulate data (15,000 observations) from the IN model to obtain its population characteristics. Table 2 contains the results. Clearly there is a big gap between most of the business cycle characteristics

\footnote{Note that per capita GDP was defined as per capita consumption plus per capita investment so as to be consistent with the model i.e. the data in Table 4 is for this definition of "GDP". Consequently, it is not what one would get if cycles in actual per capita GDP were dated.}
from the model and the data. Therefore, we might ask whether the model and data summaries are significantly different. To do this we perform a theoretical bootstrap. Simulating the model 1,000 times with a sample size equal to that of the data used in estimation, we find that it is virtually impossible to generate average expansion and contraction durations for GDP from the model that are consistent with the data. This suggests that one might want to look at whether the cycles in other series match what is in the data. For aggregate investment the average duration of expansions is 8.6 from the model, but 15.0 in the data, and simulations reveal that one cannot generate a number like the latter from the distribution of the average durations for investment expansions.

Table 2: Business Cycle Characteristics – Data and IN Model

<table>
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<th>Data</th>
<th>IN</th>
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<td>Durations (qtrs)</td>
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<td></td>
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<td>Expansions</td>
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<td>Contractions</td>
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<td>Amplitude (%)</td>
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<td>Expansions</td>
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<tr>
<td>Cumulative amplitude (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>252.0</td>
<td>80.9</td>
</tr>
<tr>
<td>Contractions</td>
<td>−3.0</td>
<td>−1.1</td>
</tr>
</tbody>
</table>

4 Pattern formation - interpretation

4.1 The Duration of a Recession and the Size of the External Finance Premium

A number of papers have now appeared reporting the evidence that recessions associated with a financial crisis are of much longer duration than normal recessions Claessens et al (2011a), (2011b), Borio (2012). Therefore it seems

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8It is also the case that the degree of synchronization of cycles in variables such a output, consumption and investment in the model are quite different than in the data.
important to ask of any model of the business cycle whether this feature is present in them. Looking at the Gilchrist et al (2009) model for the U.S. that features a financial accelerator we ask whether the duration of recessions in it depends upon the magnitude of the external finance premium ($s_t$). There are two ways this might be done. One is to relate the durations of recessions to the external finance premium. To do this we regressed the computed durations of recessions from the simulated data against the external finance premium at the beginning of each recession. While this showed a positive relationship the connection was very weak – even large changes in the premium only caused the duration to increase by a fraction of a quarter.

Following Harding(2010) an alternative indicator of the relationship between recessions and the external finance premium comes from recognizing that the recession states $R_t$ generated by BBQ (and this is also true for NBER recession indicators) follow a recursive process of the form

$$R_t = 1 - (1 - R_{t-1})R_{t-2} - (1 - R_{t-1})(1 - R_{t-2})(1 - \wedge_{t-1}) - R_{t-1}R_{t-2}\vee_{t-1}, \quad (3)$$

where $\wedge_t$ is a binary variable taking the value unity if a peak occurs at $t$ and zero otherwise, while $\vee_t$ indicates a trough. By definition $\wedge_t = (1 - R_t)R_{t+1}$ and $\vee_t = (1 - R_{t+1})R_t$. In BBQ,

$$\wedge_t = 1(\{\Delta y_t > 0, \Delta y_{t+1} < 0, \Delta y_{t+2} < 0\})$$

$$\vee_t = 1(\{\Delta y_t < 0, \Delta y_{t+1} > 0, \Delta y_{t+2} > 0\}),$$

where $\Delta_2 y_t = y_t - y_{t-2}$ will be six-monthly growth.

Now we want to compute $Pr(\vee_{t+m} = 1|\wedge_t = 1, s_{t-j})$, i.e. the probability that, conditional on the external finance premium, in $m$ periods time the economy will be at a trough, given it was at a peak at time $t$.\footnote{$j = 2$ gave the highest probabilities.} $m$ is clearly the duration of the recession. Given the binary nature of $\vee_t$ it is attractive to use functions such as the Probit form i.e. $\Phi(\alpha + \beta s_{t-j})$, where $\Phi(\cdot)$ is the cumulative standard normal distribution function. Of course there is nothing which guarantees that this is a good representation of the functional form for the probability, so it is always useful to check it against a non-parametric estimate. For large premia there are not that many observations so the non-parametric estimate may not be that reliable over the complete range of values of $s_{t-j}$.
Table 3: Probability of Recession for \( m \) Periods as External Premium Varies in GOZ Model

<table>
<thead>
<tr>
<th>External premium ( \tilde{s}_t ) (basis points)</th>
<th>( m = 2 )</th>
<th>( m = 3 )</th>
<th>( m = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0.30</td>
<td>0.18</td>
<td>0.14</td>
</tr>
<tr>
<td>305</td>
<td>0.36</td>
<td>0.21</td>
<td>0.17</td>
</tr>
<tr>
<td>481</td>
<td>0.40</td>
<td>0.23</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 3 shows what this probability is for three levels of \( s_{t-2} \) and for \( m = 3, 4, 5 \) i.e. the probability that a recession will last at least three, four or five quarters once it has begun. One might note that the non-parametric estimates are smaller than the Probit estimates over the range from 24 to 481 basis points but the same type of increase is evident. So, if a financial crisis produces long duration recessions we might expect to see the probabilities of getting longer recessions increase markedly with the level of the external finance premium. But this does not seem to be the case. Take \( m = 5 \). Whilst it is true that the probability of getting a five quarter recession increases with the level of the external finance premium, it is a very small increase. This weak effect agrees with the regression linking durations and the external finance premia noted above. Thus the Gilchrist et. al. model only delivers a relationship between recessions and the external finance premium in only a very weak way.

### 4.2 The Impact of Sentiment on Business Cycles

Consumer confidence (sentiment) is constantly invoked by media commentators and central bankers to provide an explanation or a forecast of macroeconomic outcomes. Here is one such recent statement-

"...confidence - that economists find easiest to measure. And that’s a good thing for all of us, because it’s one of the most important - an early reading of a fall in household or business confidence can help to explain why unemployment may soon start to rise, or why asset markets may become more volatile" (Gareth Hutchens, The Age, 26th Jan 2013)

Despite the statement above it is not entirely clear how easy it is to measure confidence, and most of the papers which have looked for a connection
between measured confidence and macro-economic variables have not found a very strong relationship. In a departure from this Barsky and Sims (2012) report finding an important role for confidence. They say (footnote 9, p. 1351)

"The key difference is that we focus on both short-run and medium-to longer-run links between confidence and activity. Since the short-run implications of confidence are modest, researchers who focused on the short-run found only a modest role for confidence".

Because their results are quite striking it is worth re-examining their conclusions using some different econometric techniques and concentrating upon the role of confidence in producing business cycles. We firstly look at work that Barsky and Sims did with a VAR in three variables - the levels of the log of real GDP ($y_t$), the log of real consumption ($c_t$), and a series $E5Y_t$ that measures sentiment and is constructed from a question in the Michigan Survey of Consumers. $E5Y_t$ differs from the Index of Consumer Sentiment (ICS) that is often used, but they report that the results they obtain are much the same as when ICS is used. They find very persistent (permanent) effects of confidence upon $y_t$ and $c_t$.

They estimate a recursive SVAR in $E5Y_t, y_t$ and $c_t$. In their later DSGE model they treat $E5Y_t$ as $I(0)$ (the AR(1) coefficient is about .9) but $y_t$ and $c_t$ are $I(1)$ and co-integrated. They say that imposing co-integration in the three variable case does not change their conclusions. So we will work with an SVAR in $E5Y_t, \Delta y_t$ and $ec_t = c_t - y_t$. The estimated co-integrating relation is $c_t = 1.0008y_t$, but we will replace 1.0008 by 1.0, as that is what their DSGE model assumes.

Now, because there are two $I(1)$ variables and co-integration, we will assume that there is one permanent and one transitory shock. When we add to the system the confidence measure $E5Y_t$ we introduce a new shock. This shock might be transitory but it might also have some permanent effects. Suppose we arrange the variables as $E5Y_t, y_t$, and $c_t$. Then the matrix of long-run impacts upon the variables $E5Y_t, y_t$ and $c_t$ might look like this

$$C = \begin{bmatrix} 0 & 0 & 0 \\ * & 1 & 0 \\ * & 1 & 0 \end{bmatrix}.$$ 

In this case the third shock is a transitory shock and the second is a permanent one. However, this raises the issue of how to describe the first shock that comes about due to the introduction of the $I(0)$ variable $E5Y_t$. It could be
transitory (in which case \( * = 0 \)) but it might have some permanent effects (as suggested by Barsky and Sims' work). In order to preserve co-integration it must be that the long-run effects on each of the \( I(1) \) variables are the same. Since the rank of \( C \) is normally taken to indicate the number of permanent shocks and, since it is clearly unity, when the second shock is taken to be permanent the first shock cannot be described in this way. Accordingly, following Fisher et al (2013) we will term it a mixed shock, since it can have permanent effects and comes about because the shock arises when there is a mixture of \( I(1) \) and \( I(0) \) variables. So the standard technology of permanent and transitory shocks is clearly inadequate to deal with the set-up in the Barsky and Sims paper.

Now Barsky and Sims make their SVAR recursive by putting confidence \((E5Y_t)\) first. The equation for \( E5Y_t \) will therefore be (for convenience we write this as an SVAR(1) although they make it an SVAR(3))

\[
E5Y_t = a_{11}^1E5Y_{t-1} + a_{12}^1\Delta y_{t-1} + a_{13}^1e_{ct-1} + u_{1t}. \tag{4}
\]

With a recursive system the remaining two equations of the system will be

\[
\Delta y_t = a_{21}^0E5Y_t + a_{21}^1E5Y_{t-1} + a_{22}^1\Delta y_{t-1} + a_{23}^1e_{ct-1} + \varepsilon_{2t} \tag{5}
\]

\[
e_{ct} = a_{31}^0E5Y_t + a_{32}^0\Delta y_t + a_{32}^1E5Y_{t-1} + a_{33}^1\Delta y_{t-1} + a_{33}^1e_{ct-1} + \varepsilon_{3t}. \tag{6}
\]

Figure 2 shows the impulse responses of the level of GDP to the "confidence shock" \( u_{1t} \). As Barsky and Sims note it appears that there is a permanent effect.

Barsky and Sims also estimate a DSGE model in order to look at the relative influence of "news" and "animal spirits" upon cycles. They take the innovations into \( E5Y_t \) to have three components. The first component was a permanent shock to the level of technology, the second was the innovation into an AR(1) for the growth rate in technology, and the final component was a "noise" shock. Regarding the latter they say "it can be interpreted as measurement error in the confidence data", but one would think that it might also be regarded as animal spirits since, if it is zero, then the only thing that the confidence index would depend on would be technology. In

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\( ^{10} \)When we tested the order of the VAR by a variety of criteria such as AIC and BIC all indicated that the VAR was of second order except for BIC which said it was of first order. Consequently, our empirical work is done with an SVAR(2) as that matches their DSGE model. The results are very similar with an SVAR(3).
their model they have a "noisy signal of the growth rate" of technology \((g_t)\) i.e. \(g_t\) is unobserved, and it is assumed that there is a signal \(s_t\) such that \(s_t = g_t + \varepsilon_{s,t}\). Then \(\varepsilon_{s,t}\) is taken to be the animal spirits. This seems peculiar since they have termed it a noisy signal and it would seem more sensible to call this an observation error and the residual (third) shock into confidence as animal spirits. That is the perspective we will take.

With this description the first equation of the SVAR will be

\[
E5Y_t = a_{11}^1 E5Y_{t-1} + a_{12}^1 \Delta y_{t-1} + a_{13}^1 ec_{t-1} + \rho \varepsilon_{2t} + \varepsilon_{1t},
\]

where \(\varepsilon_{2t}\) is taken to be the permanent (technology) shock. Then the second equation will have the form

\[
\Delta y_t = a_{21}^0 \Delta E5Y_t + a_{23} \Delta ec_t + a_{23}^1 \Delta y_{t-1} + \varepsilon_{2t}.
\]

The structure of this equation comes from the fact shown in Pagan and Pesaran (2008) that the error correction term appears in differenced form in any structural equation that has a permanent shock, while the difference in \(E5Y_t\) stems from our assumption that \(\varepsilon_{1t}\) has a transitory effect on the level of GDP - a feature of Barsky and Sims' DSGE model (see the impulse response in their figure 9).

Now the \(\varepsilon_{2t}\) shock can be estimated using \(ec_{t-1}, E5Y_{t-1}\) and \(\Delta y_{t-1}\) as instruments. Then, going back to the \(E5Y_t\) equation, one fits a regression using \(E5Y_{t-1}, ec_{t-1}, \Delta y_{t-1}\) and \(\hat{\varepsilon}_{2t}\) as regressors, and this recovers the "animal spirits" shock \(\varepsilon_{1t}\). In theory the remaining structural equation in the system - \(ec_t\) - can be estimated using \(E5Y_t\) and \(\hat{\varepsilon}_{2t}\) as instruments for \(E5Y_t\) and \(\Delta y_t\). However, it turns out that the instrument for \(\Delta y_t\) is quite poor, and so we cannot get a reliable estimate of the third shock \(\varepsilon_{3t}\). Nevertheless, as we are only interested in the role of confidence, we don’t need to estimate the third equation. Instead we replace it with the VAR equation for \(ec_t\)

\[
ec_t = a_{31}^1 E5Y_{t-1} + a_{32}^1 \Delta y_{t-1} + a_{33}^1 ec_{t-1} + e_{3t},
\]

and, using the relation between SVAR and VAR errors, we know that

\[
e_{3t} = b_{31} \varepsilon_{1t} + b_{32} \varepsilon_{2t} + b_{33} \varepsilon_{3t}.
\]

Therefore, since \(\varepsilon_{3t}\) is uncorrelated with \(\varepsilon_{1t}\) and \(\varepsilon_{2t}\), we can consistently estimate \(b_{31}\) and \(b_{32}\) by adding on to the regression in (9) \(\hat{\varepsilon}_{1t}\) and \(\hat{\varepsilon}_{2t}\). We are left with a shock \((b_{33}\varepsilon_{3t})\) that we cannot split up unless we can get a
good estimate of $b_{33}$, and that was not possible given the weak instruments. However in order to compute impulse responses to $\varepsilon_{1t}$ and $\varepsilon_{2t}$ we do not need to estimate the structural equation for $cc_t$. Similarly, we can generate data on $\Delta y_t$ using (7),(8) and (9). Figure 2 shows the response to the animal spirits shock when we assume it is a transitory shock into output. Clearly there are vast differences in the impulse responses.

Now Barsky and Sims estimated the parameters of the DSGE by doing indirect estimation, where the auxiliary model was a VAR in five variables - $y_t$, $c_t$, the CPI inflation rate ($\pi_t$), the three month real Treasury bill rate, $r_t$, (the expected inflation rate coming from the Michigan Survey of Consumers) and $E5Y_t$. We formulated an SVAR that captures the spirit of the DSGE model without imposing a precise structure. To do this it seemed reasonable to assume that $\varepsilon_{1t}$ and the remaining shocks in the system connected to the structural equations for $r_t$ and $\pi_t$ have a zero long-run effect on output. As we have mentioned the assumption regarding $\varepsilon_{1t}$ fits the impulse response in Barsky and Sims’ figure 9 while that for the other two shocks seems standard in most DSGE models. With these assumptions the equation for output growth becomes

$$
\Delta y_t = \alpha_1 \Delta cc_t + a_{21}^0 \Delta E5Y_t + a_{23}^0 \Delta r_t \\
+ a_{24}^0 \Delta \pi_t + a_{21}^1 \Delta y_{t-1} + \varepsilon_{2t},
$$

allowing us to estimate $\varepsilon_{2t}$ using $cc_{t-1}$, $E5Y_{t-1}$, $r_{t-1}$, $\pi_{t-1}$ and $\Delta y_{t-1}$ as instruments. Thereafter we recover $\varepsilon_{1t}$ as for the three variable SVAR.\textsuperscript{11} Figure 3 shows that the difference in impulse response of GDP to the animal spirits shock is small until we get to large horizons and so we will work with the three variable VAR in the simulations we are about to perform.

What we are fundamentally interested in is how animal spirits affect the business cycle. To assess this we need to define the cycle. Because the

\textsuperscript{11}The remaining structural equations cannot be identified without further assumptions that may be inconsistent with their DSGE model. Thus, although they use a Taylor rule that has $r_t$ depending only upon $\pi_t$ and $\Delta y_t$, the only two instruments that are available from the basic set we employ are $\hat{\varepsilon}_{1t}$ and $\hat{\varepsilon}_{2t}$, and this would effectively mean that $\hat{\varepsilon}_{1t}$ is the instrument for $\pi_t$. Their figure 9 does suggest that it could be a good instrument, thereby allowing us to estimate $\varepsilon_{3t}$. But we would still have to estimate an equation for $\pi_t$. That would need four instruments and we only have three residuals $\hat{\varepsilon}_{1t}, \hat{\varepsilon}_{2t}, \hat{\varepsilon}_{3t}$. Some extra assumption would need to be made to identify $\varepsilon_{4t}$ and it is not clear what this would be.
Barsky-Sims data is quarterly we will use BBQ to construct the statistics on the business cycle. Thus we fit 3 variable SVARs either using a recursive structure when the animal spirits are allowed to be a mixed shock i.e. they can have a permanent effect on GDP, or when the animal spirits shocks are transitory (as described above). Data is then simulated and passed through BBQ. As we have said elsewhere - Pagan and Robinson (2012) - this provides a different cut to the data and one can rarely understand business cycle outcomes from just a knowledge of impulse responses.

We do two types of simulations - one that includes all the shocks in the model and one that sets the animal spirits shocks to zero. In the recursive model there is no dependence of \( \varepsilon_{1t} \) on the other structural shocks and so we refer to the innovations in the \( E5Y \) equation as the animal spirits in that model. Table 4 shows the durations and amplitudes of the phases for both models with and without animal spirit shocks. There are some striking differences. Even though the impulse responses of GDP to animal spirits are much larger when we allow for the shocks to have permanent effects than when they are transitory, the role of these shocks in the business cycle is much larger when they are transitory. Removing them from simulations results in expansion durations being about two quarters longer when shocks are transitory. In contrast, when they are mixed (recursive model) the durations scarcely change. It is also noticeable that animal spirits are deleterious to the cycle - the complete cycle duration is shorter by 2.5 quarters when they are present i.e. there are more cycles. It is also clear that they make the amplitudes of recessions about 30% larger, and this is true of both models. So one gets a different picture of the role of sentiment here than one gets from Barsky and Sims’ reported results for their DSGE model, where animal spirits contribute virtually nothing to the fluctuations in GDP. This emphasizes the value of studying patterns when evaluating the contribution of new features that are to be built into models.

5 Pattern Prediction

Setting \( R_{t-1} = 0, R_t = 0 \) and using the recursive formula for \( R_{t+1} \) in (3) shows that \( R_{t+1} \mid R_t = \wedge_t \) is the information to be used for the forecast.\(^{12}\)

\(^{12}\)If \( R_t \) is unknown using the recursive formula for \( R_t \) and applying \( \wedge_{t-1} \wedge_t = 0 \) (since turning points must be two periods apart), we get \( R_{t+1} \mid R_t = \wedge_{t-1} + \wedge_t \), and so one needs to allow for a possible turning point at \( t - 1 \) when considering the outcome for \( R_{t+1} \).
Figure 2: Impulse Responses of GDP to Animal Spirits Shock when Shock is Treated as Mixed (3varrec) or Transitory (gdp3varlr, 5varlr)
Table 4: Business Cycle Statistics in Two Models with and Without Animal Spirit Shocks

<table>
<thead>
<tr>
<th></th>
<th>Transitory With</th>
<th>Transitory Without</th>
<th>Recursive Mixed With</th>
<th>Recursive Mixed Without</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dur Recession</td>
<td>3.6</td>
<td>3.4</td>
<td>4.0</td>
<td>3.4</td>
</tr>
<tr>
<td>Dur Expansion</td>
<td>25.7</td>
<td>28</td>
<td>19.2</td>
<td>19.4</td>
</tr>
<tr>
<td>Amp Recession</td>
<td>-1.8</td>
<td>-1.2</td>
<td>-2.0</td>
<td>-1.4</td>
</tr>
<tr>
<td>Amp Expansion</td>
<td>25.2</td>
<td>25.9</td>
<td>17.7</td>
<td>16.8</td>
</tr>
</tbody>
</table>

Hence $E(R_{t+1}| R_t, z_t) = E(\wedge_t | z_t)$. Then, from the definition of $\wedge_t$ (for BBQ) as

$$\wedge_t = 1(\{\Delta y_t > 0, \Delta_2 y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\})$$

$$= 1(\Delta y_t > 0)1(\Delta_2 y_t > 0)1(\Delta y_{t+1} < 0)1(\Delta_2 y_{t+2} < 0),$$

we have

$$\Pr(R_{t+1} = 1|R_t, z_t) = E\{1(\Delta y_t > 0)1(\Delta_2 y_t > 0)1(\Delta y_{t+1} < 0)1(\Delta_2 y_{t+2} < 0)|z_t\} \leq E(1(\Delta y_{t+1} < 0)1(\Delta_2 y_{t+2} < 0)|z_t) \tag{10}$$

$$\leq E(1(\Delta y_t < 0)|z_t). \tag{11}$$

Equation (10) points to the fact that predicting a recession involves successfully predicting negative quarterly and six-monthly growth over the two quarters following on from the prediction point, (assuming of course that it is known that an expansion held at $t$ and $t-1$). Equations (11) and (12) are useful for providing some upper bounds to the probability of predicting a recession given any set of information. The last result is particularly useful as it is extremely simple to compute. Moreover, the ability to predict a negative growth rate in activity is common to virtually all definitions of a recession. If one cannot predict negative growth then one won’t be able to predict recessions, as it represents an upper bound to $\Pr(R_{t+1} = 1|R_t, z_t)$.

It might be asked why one didn’t just fit a discrete choice model such as Probit using $R_{t+1}$ as the dependent variable and $R_t, R_{t-1}$ as independent
variables? As discussed in Harding and Pagan (2011) this cannot be done due to the restriction that phases must be at least two quarters in duration. It is not unusual to see Probit fits using $R_{t+1}$ as the dependent variable but with (at most) $R_t$ as an independent variable. However, the use of $R_t$ as a regressor is illegitimate, as it is not a pre-determined variable, being constructed from future data - see Harding and Pagan (2011). One can compute a non-parametric solution by simply conditioning on the sample of observations that have $R_t = 0, R_{t-1} = 0$, but, for non-parametric methods to work, $z_t$ must be of small dimension.\(^\text{13}\) It seems likely however that any model will imply a substantial number of variables in $z_t$.

Turning to the bounds we can see that they are more straightforward to compute. If the model implies that $\Delta y_{t+1}$ is normally distributed around $E(\Delta y_{t+1}|z_t) = z'_t \beta_1$ with variance $\sigma^2$, then $E(1(\Delta y_{t+1} < 0)|z_t) = \Phi(-z'_t \beta_1 / \sigma)$, where $\Phi$ is the cumulative standard normal density function. Similarly, if $E(\Delta y_{t+1}|z_t) = z'_t \beta_1$ and $E(\Delta_{2y_{t+2}}|z_t) = z'_t \beta_2$, then we can find the requisite probability in (11) from a bivariate normal with mean zero and covariance matrix given by the covariance of the errors for $\Delta_{2y_{t+2}}$ and $\Delta y_{t+1}$. Of course these linear forms will be appropriate if $\Delta y_t$ follows a VAR process. This is true of most macroeconomic models that are constructed. There is however a complication in that the macroeconomic models often contain latent variables, and it is the joint vector of latent and observable variables which follow a VAR. Restricting attention to fitting a VAR to the observables alone may mean that it will be of higher order than that which includes the latent variables, although omission of variables need not always increase the order needed for the VAR. Now the GOZ model is a VAR(2) in all its variables but, since some of these are latent, it will be necessary to trial a higher order VAR to compute the relations between $\Delta y_{t+1}, \Delta_{2y_{t+2}}$ and the observable model variables.

Table 5 below performs an exercise with data on nine of the ten variables used by GOZ in estimation. There are actually two missing series that they used for estimation and we were not able to get. One of these was the degree of leverage and the other the external finance premium. These came from some micro data sets. We substituted the BAA spread for the external finance premium but had to ignore the degree of leverage, and hence it effectively becomes a latent variable.\(^\text{14}\) If these two series had been available

\(^{13}\) We computed $\Pr(R_t = 1| R_{t-1} = 0, R_{t-2} = 0, \tilde{q}_t)$ earlier using this method.

\(^{14}\) The Baa spread used is that available at the beginning of the quarter for which a
we could have computed $E(\Delta y_{t+1} < 0 | z_t)$ and $E(\Delta_2 y_{t+2} < 0 | z_t)$ with the Kalman predictor.

Table 5: **First Period Probabilities of Recession Events and Predicted Growth Signs Using the GOZ Observables Fitted to a VAR(4)**

<table>
<thead>
<tr>
<th>Peak</th>
<th>$1(\Delta y_{t+1} &lt; 0)1(\Delta_2 y_{t+2} &lt; 0)$</th>
<th>$1(\Delta y_{t+1} &lt; 0)$</th>
<th>$\Delta y_{t+1}$</th>
<th>$\Delta_2 y_{t+2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1977/3</td>
<td>.25</td>
<td>.36</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>2 1978/4</td>
<td>.52</td>
<td>.59</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3 1981/1</td>
<td>.85</td>
<td>.87</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4 1990/2</td>
<td>.20</td>
<td>.32</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>5 2000/4</td>
<td>.46</td>
<td>.56</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6 2002/3</td>
<td>.35</td>
<td>.50</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>7 2007/4</td>
<td>.32</td>
<td>.47</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

We used the 15,000 observations from the simulations in section 3 to estimate the parameters $\beta_1$ and $\beta_2$ above. This was done by fitting VAR’s to the simulated data using just what would be the observed variables. VAR’s were fitted up to order five. By that order the probabilities of a recession seemed to have stabilized. Table 5 then shows these. This is done by computing the expectations in Equations (10) and (11) when $t$ corresponds to a peak in the data. We refer to these as "first period" probabilities as they are predictions of the first period of a recession. To be more specific, take the peak occurring in 1977/3. Since the recession started in 1977/4 we are therefore forming a prediction in 1977/3 about 1977/4 (of course this forecast is predicated on still being in an expansion in 1977/3). We need to emphasize that the cycle here is in *per capita GDP* and so there may be more recessions, as well as a difference in timing, to those presented by the NBER, which essentially relate to the *level* of economic activity.

Bearing in mind that the unconditional probability of a recession over this period is .27, that current data on items like consumption and investment would not have been available when a prediction was made, and that the table gives *upper bounds* to the probability of a recession, at best it would seem that prediction is to be made.
the GOZ model would have predicted two of the seven recessions. The biggest success would have been the second of the famous double dip recessions in the late 1970s and early 1980s, although it should be noted that, for the years leading up to 1981, the model predicts very high probabilities of a recession, reaching .90 (1980/1) and .99 (1980/2). Consequently, there were quite a few false predictions.\footnote{Of course these are upper bounds and Pr(R\textsubscript{t+1}|R\textsubscript{t}, z\textsubscript{t}) might be much smaller. But if so then it would seem that the model would fail to predict recessions at all.} The table also shows the signs predicted for the future quarterly and six-monthly growth rates that distinguish a recession.

Equation (11) has the implication that, if the information \(z_t\) is to be useful in predicting recessions, it must be correlated with future shocks. A quick check on whether a model would be able to predict such a quantity is to ask how important the unpredictable part of future shocks are to these growth outcomes. Shocks, such as technology, often have an autoregressive structure, and it is the innovation (the unpredictable part) whose impact upon the business cycle is to be determined. We therefore simulate the GOZ model turning off the contemporaneous innovations. That is, the model is run with the current innovations set to zero, although they are re-set to their actual values in later periods. To illustrate what is done, take an AR(1) \(\xi_t = \rho \xi_{t-1} + e_t\), where \(e_t\) is white noise. Defining \(\bar{\xi}_t = \rho \xi_{t-1}\), we note that \(\xi_t\) and \(\bar{\xi}_t\) differ only in that the current innovation is set to zero; in other words, \(\bar{\xi}_t\) is the predicted value of \(\xi_t\) using information at \(t - 1\).

Table 6 shows business cycle characteristics from the GOZ model with current innovations present (equivalent to basing the computation on \(\xi_t\)) and with them suppressed (equivalent to \(\bar{\xi}_t\), and hence designated GOZ\(^{-}\)). It is clear that the innovations have a substantial effect upon the average cycle characteristics. Without these innovations, expansions become very long, and so there will be fewer recessions. Because these innovations are so important, and they are what makes \(\Delta y_{t+1}\) differ from \(E(\Delta y_{t+1}|z_t)\) in the analysis above, we can see why the GOZ model had limited success when it came to predicting recessions (on average). Repeating the exercise with the IN model one finds that, just as with the GOZ model, expansion durations double once current shocks are excluded. Thus this simple strategy of re-running the model with the innovations removed seems a good way to assess the likelihood of successful recession prediction in those cases where the assumptions such as normality needed to compute the probabilities in Table 5 do not hold.\footnote{The IN model is an example of this as it was solved using a second order approximation,}
Table 6: Impact of Current Shocks on Business Cycles in GOZ Model

<table>
<thead>
<tr>
<th></th>
<th>GOZ</th>
<th>GOZ⁻</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Durations (qtrs)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>14.8</td>
<td>33.9</td>
</tr>
<tr>
<td>Contractions</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td><strong>Amplitude (%)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>9.0</td>
<td>15.9</td>
</tr>
<tr>
<td>Contractions</td>
<td>−1.6</td>
<td>−0.8</td>
</tr>
<tr>
<td><strong>Cumulative amplitude (%)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>107.9</td>
<td>461.8</td>
</tr>
<tr>
<td>Contractions</td>
<td>−5.2</td>
<td>−2.8</td>
</tr>
</tbody>
</table>

6 Conclusion

Patterns have long been noticed and commented on. One often see comments like “business cycles are of duration of 5-8 years” and “financial crises lead to deep and long-lasting recessions”. But, with the exception of clustering (via GARCH and factor models) the econometrics of patterns has been relatively undeveloped. Computational advances are now allowing us to develop an econometrics for patterns and to be able to use patterns to provide interesting information for evaluating our models. The lecture has tried to summarize work over the past decade on the econometrics of patterns and to show how this work can be used to throw light on our ability to model macroeconomic and financial linkages. For prediction of patterns it is necessary that one be able to precisely describe the way that patterns relate to history and we have looked into this for the business cycle although the approach extends more generally.

7 References

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