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Game Theory
Winter term 2008-2009

Problem Set 9

(SPNE in Finitely Often and Infinitely Often Repeated Games)

Name: _ _ _ _ _ _ _ _ _ _ _ _

9.1 The following simultaneous-move game is played twice, with the outcome of the first stage observed before the second stage begins. The payoffs of the repeated game are the sum of the payoffs of both stages; that is, there is no discounting.

	P₂	**Q₂**	**R₂**	**S₂**
P_1	2, 2	x, 0	-1, 0	0, 0
Q_1	0, x	4, 4	-1, 0	0, 0
R_1	0, 0	0, 0	0, 2	0, 0
S_1	0,-1	0,-1	-1,-1	2, 0

The variable x is greater than 4 ($x > 4$), so that (Q_1,Q_2) is not an equilibrium in the one-shot game. For what values of x is the following strategy (played by both players) a subgame perfect equilibrium?

Play Q_i in the first stage. If the outcome in the first stage is

- -- (Q_1, Q_2), play P_i in the second stage,
- -- (y, Q_2), where $y \neq Q_1$, play R_i in the second stage,
- -- (Q_1, z), where $z \neq Q_2$, play S_i in the second stage,
- -- (y, z), with $y \neq Q_1$ and $z \neq Q_2$ play P_i in the second stage.

9.2 Now assume that the following simultaneous-move game is played twice, with the outcome of the first stage observed before the second stage begins. Again, there is no discounting.

	a_2	b_2	c_2
a_1	3, 1	0, 0	5, 0
b_1	2, 1	1, 2	3, 1
c_1	1, 2	0, 1	4, 4

Can the payoff combination (4, 4) be obtained in the first stage of a subgame perfect equilibrium in pure strategies? If so, specify the complete strategies of both players. If not, prove why not.

9.3 **(Prisoners' Dilemma 1)** Consider the following version of the Prisoners' Dilemma game:

	$\mathbf{C_2}$	$\mathbf{D_2}$
$\mathbf{C_1}$	5, 5	-10, 10
$\mathbf{D_1}$	10, -10	-5, -5

The basic game: Prisoners' Dilemma

This basic game is played once in the first stage, with the outcome observed by both players. Nature decides whether the game is continued (probability *p*) or not (probability *1-p*). If the game continues, the first stage is repeated until Nature ends the game. A player's total payoff is the (undiscounted) sum of the payoffs over all stages played.

Consider the following strategy of player *i (=1, 2): "Play* C_i *in the first round. In every subsequent round play* C_i*, if and only if the outcome of all rounds before has been* (C_1, C_2)*. Otherwise play* D_i*."* For what values of *p* does this strategy (played by both players) constitute a subgame perfect equilibrium?

9.4 **(Prisoners' Dilemma 2)** Consider the following version of the Prisoners' Dilemma:

	C_2	D_2
C_1	x, x	-10, 10
D_1	10, -10	-5, -5

The basic game: Prisoners' Dilemma

This basic game is played once in the first stage, with the outcome observed by both players. Then a coin flip decides whether the game is continued (with probability ½) oder not (with probability ½). If the game continues, the first stage is repeated until Nature ends the game. Again a player's total payoff is the (undiscounted) sum of the payoffs over all stages played.

Consider the following strategy of player *i (=1, 2): "Play* C_i *in the first round. In every subsequent round play* C_i*, if and only if the outcome of all rounds before has been* (C_1, C_2)*. Otherwise play* D_i*."* For what values of x does this strategy (played by both players) constitute a subgame perfect equilibrium?

9.5 **(Collusion in Cournot Duopoly)** In a homogeneous products market with inverse demand given by $P(a) = max \{10 - x , 0\}$ two firms $(i = 1, 2)$ compete by simultaneously choosing output quantities a_1 and a_2 in each period. The total output a is given by $a = a_1 + a_2$. Both firms face the same cost function $C(a_i) = a_i$. The probability that the market disappears is 40% in each round. There is no discounting. The payoff of the firms is simply their profit.

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a) The two firms are sick of competion. They agree *(i)* that each of them will produce half of the monopoly output in the next period, *(ii)* that they will produce the same quantity in every subsequent period, given that both firms have been co-operative so far, and *(iii)* that they will return to producing the same output as before the agreement if at least one firm deviates in at least one round. Will the firms stick to the agreement?

b) Now assume that the firms agree *(i)* that both will produce the quantity $a_i = 2.5$ in the next period, *(ii)* that they will produce the same quantity in every subsequent period, given that both firms have been co-operative so far, and *(iii)* that they will return to producing the same output as before the agreement if at least one firm deviates in at least one round. Will the firms stick to the agreement?