Rudolf Kerschbamer

Commitment and Information in Games

Problem Set 8

(SPNE in Dynamic Games with Continuous Action Spaces and Imperfect Information)

Name: _ _ _ _ _ _ _ _ _ _ _ _

8.1 **(Strategic Trade Subsidies)** Consider the two-stage subsidy game described in Example 30 of Lecture 8.

a) Write down the payoffs of the four players as functions of the actions taken by them: $u_x(a_x, a_y, a_1, a_2)$, $u_y(a_x, a_y, a_1, a_2)$, $u_1(a_x, a_y, a_1, a_2)$ and $u_2(a_x, a_y, a_1, a_2)$. Is the simultaneous move game of stage two (for given (a_x, a_y)) a game of positive externalities or a game of negative externalities? Is this game one of strategic substitutes or of strategic complements?

b) Find the stage-two NE for $a_x = a_y = 0$. Derive the associated payoffs of the four players. Sketch the reaction functions, the NE and the indifference curves through the NE in a diagram (a_1 on the x-axis and a_2 on the y-axis).

c) Assume now that country x is able to precommit to an excise subsidy of $a_x \geq 0$ per unit of output of firm 1, while country y is (for some exogenous reason) restricted to announce $a_y = 0$. Solve for the stage-two NE for any $a_x \geq 0$ and $a_y = 0$ and then for the SPNE of the whole game. Derive the payoffs of the four players associated with the SPNE of the game and compare them with the payoffs in question b). Sketch the new equilibrium in the diagram of question b) and explain the result.

d) Solve now for the two stage game as described in Example 30. That is, first derive stage-two best-response functions for any (a_x, a_y); then derive the stage-two NE for any (a_x, a_y); now write down the reduced form payoff functions of players x and y; and then solve for the stage-one NE. How do the SPNE strategies of the four players look like, what is the subgame perfect outcome of the game? Derive the payoffs of the four players associated with the SPNE of the game and compare them with the payoffs in question b). Explain the result.

8.2 **(Stackelberg Cournot Oligopol)** In a homogeneous goods market with the inverse demand function $P(x) = max \{10\text{-}x , 0\}$ three firms $(i = 1, 2, 3)$ compete by choosing output quantities a_1, a_2 and a_3, whereas $x = a_1 + a_2 + a_3$. Firm 1 can precommit to a quantity first. Firms 2 and 3 observe a_1 and then simultaneously choose a_2 and a_3. All firms have the same cost function $C(a_i) = 2a_i$.

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a) For each a_1 solve for the stage-two best response functions and for the stage-two NE.

b) Solve for the optimal a_1 assuming that firm 1 correctly anticipates the other two firms' reactions (it is unique). How does the SPNE look like, how does the subgame perfect oucome look like?

c) How does the subgame perfect equilibrium behavior of firms *1* and *2* change when firm *3* leaves the market? Are firms *1* and *2* producing more or less than before?