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Commitment and Information in Games

Problem Set 7

(SPNE in Dynamic Games with Continuous Action Spaces and Perfect Information)

Name: _ _ _ _ _ _ _ _ _ _ _ _

7.1 **(Stackelberg-Bertrand Duopoly with Heterogeneous Products)** Consider the Stackelberg-Bertrand Duopoly described in Example 26 of Lecture 7. Assume that g = 0.8 and c = 0.2.

a) Write down the payoff functions $u_1(a_1, a_2)$ and $u_2(a_1, a_2)$ of the two firms. Is this a game of positive externalities or a game of negative externalities? Is this a game of strategic substitutes or a game of strategic complements?

b) Find the follower's (pure strategy) best response function $b_2(s_1)$. Is the slope of this function positive or negative? Explain why.

c) Assume that the leader correctly anticipates the follower's reaction. Solve for the leader's optimal action.

d) Write down the SPNE of the game (it is unique).

e) Solve for the subgame perfect outcome of the game. Derive the SPNE quantities and SPNE profits. Compare the SPNE profits of this two-stage game with the NE profits of the simultaneous-move version of the game (Example 15 in Lecture 5). In particular, compare *(i)* the profit of the leader in the SPNE of the two-stage version of the game with the profit of the leader in the NE of the simultaneous-move version of the game; *(ii)* the profit of the follower in the SPNE of the two-stage version of the game with the profit of the follower in the NE of the simultaneous-move version of the game; and *(iii)* the profit of the leader in the SPNE of the two-stage version of the game with the profit of the follower in the SPNE of the two-stage version of the game. Is there are first mover or a second mover advantage in this game?

f) Assume now that g = -0.8 (in words: minus point 8) and c = 0.2. Answer questions a and b for this parameterization. Try to answer the following question without solving the game: Is there a first mover or a second mover advantage in this game? Give a justification for your answer.

7.2 **(Stackelberg-Cournot Duopoly with Heterogeneous Products)** Consider the Stackelberg-Cournot Duopoly described in Example 27 of Lecture 7. Assume that g = 0.8 and c = 0.2.

a) Write down the payoff functions $u_1(a_1, a_2)$ and $u_2(a_1, a_2)$ of the two firms. Is this a game of positive externalities or a game of negative externalities? Is this a game of strategic substitutes or a game of strategic complements?

b) Find the follower's (pure strategy) best response function $b_2(s_1)$. Is the slope of this function positive or negative? Explain why.

c) Assume that the leader correctly anticipates the follower's reaction. Solve for the leader's optimal action.

d) Write down the SPNE of the game (it is unique).

e) Solve for the subgame perfect outcome of the game. Derive the SPNE prices and SPNE profits. Compare the SPNE profits of this two-stage game with the NE profits of the simultaneous-move version of the game (Example 16 in Lecture 5; see one of the exercises in Problem Set 5). In particular, compare *(i)* the profit of the leader in the SPNE of the two-stage version of the game with the profit of the leader in the NE of the simultaneous-move version of the game; *(ii)* the profit of the follower in the SPNE of the two-stage version of the game with the profit of the follower in the NE of the simultaneous-move version of the game; and *(iii)* the profit of the leader in the SPNE of the two-stage version of the game with the profit of the follower in the SPNE of the two-stage version of the game. Is there are first mover or a second mover advantage in this game?

f) Assume now that $g = -0.8$ (in words: minus point 8) and c = 0.2. Answer questions a and b for this parameterization. Try to answer the following question without solving the game: Is there a first mover or a second mover advantage in this game? Give a justification for your answer.

7.3 **(Rotten Kid Theorem)** In a three person household (father, mother, child) the parents not only care about their own well-being but also about the happiness of their child. The child, on the other hand, is completely egoistic. With its work effort the child can increase the family income. An effort *a* increases the family income by *20a* whereas the child incurs a disutility of a^2. The parents observe the child's effort and then decide about the child's pocket money of amount *b*. The child's preferences are given by the utility function

$$U_C(a, b) = ln\,(100 - a^2 + b)$$

the parents' utility function is

$$V_P(a, b) = U_C(a, b) + U_P(a, b)$$

with

$$U_P(a, b) = ln\,(400 + 20a - b).$$

a) Show that in the unique subgame perfect equilibrium the egoistic child chooses its effort *a* such that the familiy income less the child's disutility is maximized.

b) Now assume that the parents are completely egoistic [$V_P(a, b) = U_p(a, b)$ instead of $V_p(a, b) = U_C(a, b) + U_P(a, b)$] and find the subgame perfect equilibrium. Does the equilibrium effort *a* still maximize the familiy income less the child's disutility of effort?

[Further Reading: The *Rotten Kid Theorem* originates from: Becker, G. (1974), A Theory of Social Interaction, *Journal of Political Economy* 82, 1063-1093. Interesting is also: Bergstrom, T. (1989), A Fresh Look at the Rotten Kid Theorem and Other Haushold Mysteries, *Journal of Political Economy* 97, 1138-1159; and Cornes, R. and E. Silva (1999), Rotten Kids, Purity, and Perfection, *Journal of Political Economy* 107, 1034-1040. Find additional literature below]

7.4 **(Samaritan's Dilemma)** In a three person household (father, mother, child) the child is not only interested in its own welfare but also in the well-being of its parents. The parents however are completely egoistic. During their working life (period 1) they decide which fraction *a* of their total income of € 400 they want to save for their retirement (period 2). The child observes the amount of savings *a* and decides (in period 2) which amount *b* of its total income of € 200 to contribute to the maintenance of its parents. The preferences of the parents are given by the utility function

$$U_P (a, b) = ln (400 - a) + ln (a + b)$$

The child's utility function is

$$V_C (a, b) = U_C (a, b) + U_P (a, b)$$

with

$$U_C (b) = ln (200 - b).$$

a) Show that in the unique subgame perfect equilibrium the parents make an inefficient saving decision. [Hint: show that both generations would be strictly better off if the child could commit itself in period one – before the partents' saving decision is made – to contribute exactly the amount it would contribute in the subgame perfect equilibrium of the original game.]

b) Now assume that the child behaves completely egoistically as well [$V_C (a, b) = U_C (a, b)$ instead of $V_C (a, b) = U_C (a, b) + U_P (a, b)$] and solve for the subgame perfect equilibrium. Is the parents' equilibrium saving decision efficient?

[Further Reading: The *Samaritan's Dilemma* is due to Buchanan, J. (1975), The Samaritan's Dilemma, in E. Phelps (ed.) *Altruism, Morality, and Economic Theory*, Sage Foundation, New York, 71-85. Interesting is also: Lindbeck, A. und L. Weibull (1988), Altruism and Time Consistency: The Economics of Fait Accompli, *Journal of Political Economy* 96,1165-1182. *Rotten Kid* as well as *Samaritan* are discussed in: Bruce, N. und M. Waldman (1991), The Rotten Kid Theorem meets the Samaritan's Dilemma, *Quarterly Journal of Economics* 105, 155-165; and Dijksta, B. (2000), Samaritan vs Rotten Kid: Another Look, Mimeo, Universität Heidelberg (available online).]