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Commitment and Information in Games

Problem Set 6

(Subgame Perfect Equilibrium in Finite Games)

Name: _ _ _ _ _ _ _ _ _ _ _

6.1 **(Tom and Jerry)** Jerry (she) can hide in the kitchen, the bathroom or in the bedroom. Tom (he) can look for her at only one of these places. If he searches at the right place, he finds her and wins. If he searches at the wrong place, he loses and Jerry wins.

a) Assume that Tom can observe where Jerry hides before he starts searching. Represent this situation as a normal form and as an extensive form game. Solve for all **Nash equilibria** (**NEs**) in pure strategies and all **subgame perfect Nash equilibria** (**SPNEs**).

b) Now assume that Jerry knows where Tom will look for her before she hides. Represent this situation as a normal form and as an extensive form game. Solve for all NEs in pure strategies and all SPNEs.

c) Represent the situation where neither Tom nor Jerry can observe what his/her partner does as a normal form and as an extensive form game. Solve for all NEs in pure strategies and all SPNEs.

6.2 **(Bartók and Schubert)** Player 1 (he) considers whether to spend a quiet evening at home or to attend a classical concert with a friend (player 2, she). Works of Bartók (B) and Schubert (S) are played. If player 1 chooses to stay at home (H) the game ends with payoffs *(2, 2)*. If he chooses the concert (C), both players simulaneously have to decide between Bartók and Schubert. If both choose B, he receives a payoff of *3* and she a payoff of *1*. If both choose S, he gets 1 and she gets 3. If they choose different concerts, both receive a payoff of zero.

a) Represent this situation as a normal form and as an extensive form game (in the normal form he is the row player, she is the column player).

b) Solve for all SPNEs in pure and mixed strategies.

6.3 **(Cookie Game)** Adam and Berta are bargaining over how to split *7* (indivisible) cookies. Their bargaining is represented by the following two-stage game: In the first stage, Adam claims a number of cookies for himself (out of the set *{0, 1, 2, 3, 4, 5, 6, 7}).* In the second stage, Berta either accepts or declines. If Berta accepts, the cookies are split according to Adam's suggestion. If she declines, each of them gets a single cookie, the remaining cookies are destroyed.

a) Represent this situation as an extensive-form game (in the representation assume that each players utility payoff is equal to the number of cockies he/she gets).

b) Solve for all SPNEs in pure strategies.

c) Are there any (not necessarily subgame perfect) NEs in which Adam claims one cookie for himself and in which Berta accepts? If so, list all combinations of strategies leading to this outcome. If not, prove why not.

6.4 **(Twice Repeated Game 1)** Consider a game in which the following simultaneous-move game is played twice, with the outcome of the first stage observed before the second stage begins. The payoffs of the repeated game are the sum of the payoffs of both stages; that is, there is no discounting.

s_2 / s_1	**L**	**R**
U	1, 1	0, 0
D	5, 2	1, 1

a) Represent this situation as an extensive-form game.

b) How many SPNEs does this game have? How do they look like (specify the equilibrium strategies)?

6.5 **(Twice Repeated Game 2)** Consider a game in which the following simultaneous-move game is played twice, with the outcome of the first stage observed before the second stage begins. The payoffs of the repeated game are the sum of the payoffs of both stages; that is, there is no discounting. How many SPNEs does this game have? Specify the equilibrium strategies played in some of them.

s_1 \ s_2	**L**	**R**
U	1, 1	0, 0
D	0, 0	2, 2

6.6 **(An Abstract Game)** Consider a game with two players, player 1 and player 2, where player 1 can choose an action a_1 from a finite set A_1 that contains x_1 elements, while player 2 can choose an action a_2 from a finite set A_2 that contains x_2 elements. Player i *(=1, 2)'*s payoff if the action choices are (a_1, a_2) is $u_i(a_1, a_2)$.

a) Suppose that the two players move simultaneously. How many pure strategies does each player have?

b) Now suppose that player 1 moves first and that player 2 observes player 1's move before choosing her move. How many pure strategies does each player now have?

c) Suppose the game in (b) has multiple SPNEs. Show that this implies that there must exist two pairs of moves (a_1, a_2) and (a'_1, a'_2) where either $a_1 \neq a'_1$ or $a_2 \neq a'_2$ such that either

$$\text{(i)} \quad u_1(a_1, a_2) = u_1(a'_1, a'_2)$$

or

$$\text{(ii)} \quad u_2(a_1, a_2) = u_2(a'_1, a'_2)$$