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Commitment and Information in Games

Problem Set 5

(Nash Equilibrium in Infinite Games)

Name: _ _ _ _ _ _ _ _ _ _ _ _

5.1 **(The Tragedy of the Commons Version 2)** Two farmers graze their goats on the village green. Denote the number of goats farmer i (= 1, 2) owns by g_i and the total number of goats on the green by $G = g_1 + g_2$. The cost of buying and caring for a goat is constant at 4, independently of how many goats a farmer owns. The value to a farmer of grazing a goat on the green when a total of G goats are grazing is $v(G) = max\{10 – G/2, 0\}$. During the spring the farmers simultaneously choose how many goats to own. Assume goats are continuously divisible and that each farmer affords no more than 20 goats.

a) Represent this situation as a normal-form game. In the representation assume that each farmer's payoff is simply her profit.

b) Find the (pure strategy) best response correspondences $B_1(g_2)$ and $B_2(g_1)$ of the two farmers. Is this a game of strategic substitutes or a game of strategic complements? Explain why.

c) Find the (pure strategy) Nash equilibrium (it is unique). Is the equilibrium outcome efficient? Why (why not)?

d) Plot the best response correspondences of the two farmers (cut them at $g_i = 10$) and the NE in the diagram below (plot g_1 on the x axis and g_2 on the y axis). Also sketch the indifference curves of the two farmers through the NE (i.e., sketch the locus of points that yield exactly the same profit as the NE outcome). Which points in the figure Pareto dominate the NE outcome?

5.2 **(Cournot Duopoly with Differentiated Products)** Consider the Cournot Duopoly described in Example 16 of Lecture 5.

a) Represent this situation as a normal-form game. In the representation assume that each firm's payoff is simply its profit.

b) Find the (pure strategy) best response correspondences $B_1(s_2)$ and $B_2(s_1)$. Is this a game of strategic substitutes or a game of strategic complements? Explain why.

c) Find the (pure strategy) Nash equilibrium (it is unique).

5.3 **(Bertrand Duopoly with Differentiated Products)** Two firms $(i = 1, 2)$ selling differentiated products simultaneously choose prices p_1 and p_2 in [0, 2]. Total market size is 1. There are no fixed costs of production, marginal costs are constant at $c = 0.2$. If $p_j - p_i > 1$ then firm i captures the whole market: $D_i(p_1, p_2) = 1$ and $D_j(p_1, p_2) = 0$. If $|p_2 - p_1| \leq 1$ then firm i's demand is given by $D_i(p_1, p_2) = ½ + (p_j – p_i)/2$.

a) Represent this situation as a normal-form game. In the representation assume that each firm's payoff is simply its profit.

b) Find the (pure strategy) best response correspondences $B_1(p_2)$ and $B_2(p_1)$ and plot them on a 2x2 square (firm 1's strategy on the x-axis and player 2's strategy on the y-axis). Is this a game of strategic substitutes or a game of strategic complements? Explain why.

c) Find the (pure strategy) NE (it is unique) and display it on the diagram of the previous page. Sketch the indifference curves of the two firms through the NE (i.e., sketch the locus of points that yield exactly the same profit as the NE outcome). Which points in the figure Pareto dominate the equilibrium outcome?

5.4 **(Cournot Oligopoly with Homogeneous Products)** Consider a market populated by n firms who compete by simultaneously choosing their output quantities. Let s_i denote the quantity produced by firm i, and let $x = s_1 + ... + s_n$ denote the aggregate quantity on the market. Inverse demand is given by $P(x) = max\{1 - x/2, 0\}$. There are no fixed costs of production and the marginal cost is constant at c = ¼ .

a) Represent this situation as a normal-form game. In the representation assume that each firm's payoff is simply its profit.

b) Find the (pure strategy) best response correspondences for each firm i.

c) Find all (pure strategy) Nash equilibria (use the symmetry of the game).

5.5 **(Divide the Dollar Version 2)** Two players simultaneously make demands to divide € 100. Simultaneously here means that each player *i* privately writes his demand $s_i \in [0, 100]$ on a sheet of paper, not knowing what the second player has written. An impartial arbitrator collects both sheets and calculates the sum of the claims. If the sum is smaller than or equal to € 100, each player *i* receives his demand s_i; otherwise, both players get € 10. Claims can be made in real numbers (each € is perfectly divisible).

a) Represent this situation as a normal-form game. In the representation assume that each player's utility payoff is simply equal to his monetary payoff.

b) Find the (pure strategy) best response correspondences $B_1(s_2)$ and $B_2(s_1)$.

c) Find all (pure strategy) Nash equilibria.