Rudolf Kerschbamer

Commitment and Information in Games

Problem Set 4

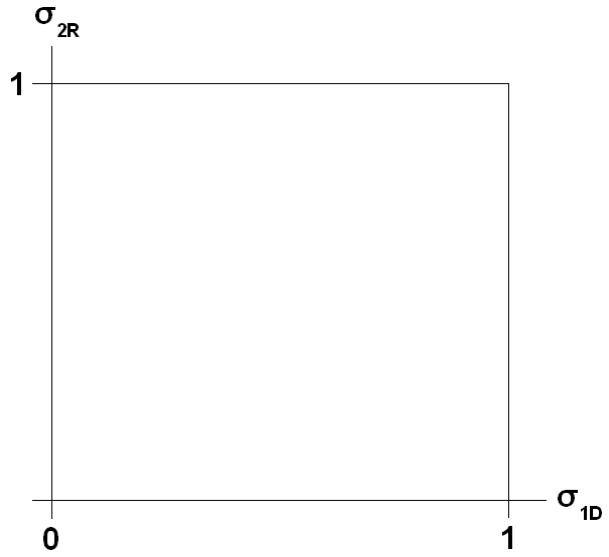
(Mixed Strategy Nash Equilibrium in Finite Games)

Name: _ _ _ _ _ _ _ _ _ _ _

4.1 Find and plot *(i)* the mixed best-response correspondences $\tilde{B}_1(\sigma_2)$ and $\tilde{B}_2(\sigma_1)$ of both players, and *(ii)* all Nash equilibria in pure and mixed strategies of the following normal-form games. [Hint: Define the row player's mixed strategy to be the probability of choosing D, denote this probability by σ_{1D} and display it on the x-axis; similarly, define the column player's mixed strategy to be the probability of choosing R, denote this probability by σ_{2R} and display it on the y-axis]

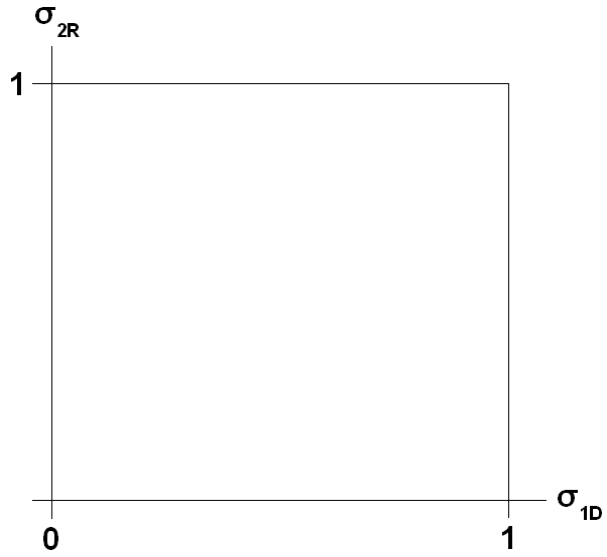
a) Here is the first game:

s_1 \ s_2	**L**	**R**
U	1, 1	0, 0
D	5, 2	1, 1



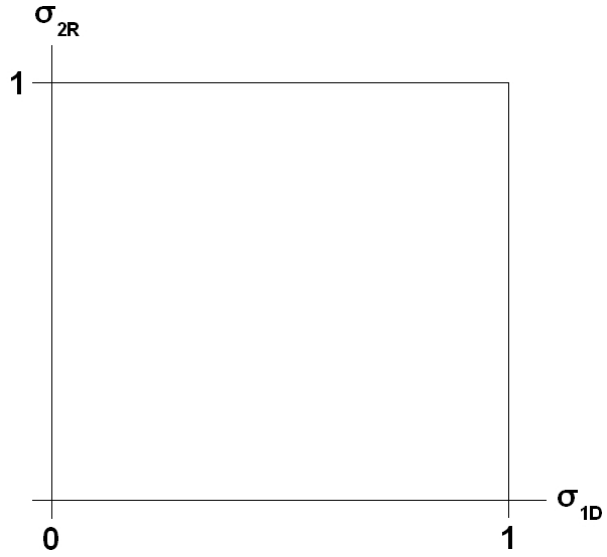
b) And here is the second game:

s_2 / s_1	**L**	**R**
U	0, 3	2, 2
D	10, 1	-1, 4



c) And here is the third game:

s_1 \ s_2	**L**	**R**
U	1, 1	0, 2
D	2, 0	-1, -1



4.2 **(Battle of the Sexes)** While at separate workplaces, Oscar and Tina must (simultaneously) decide whether to go to the football stadium or to the cinema. Both would rather spend the evening together than apart (the payoff from spending the evening apart is -2 for each player), but Oscar is a cinema fan (payoff of +5 for him and +2 for her when they spend the evening together at the cinema) while Tina is a football fan (payoff of +6 for her and +1 for him when they spend the evening together at the football stadium).

a) Represent this situation as a normal form game.

b) Solve for all Nash equilibria in pure and mixed strategies.

4.3 **(Chicken Game)** In a group of teenagers, two guys fight for the position of the gang leader. The dispute is settled by a trial of courage: both drive with their cars towards each other along a street that is too narrow for the cars to pass safely unless at least one driver slows down. If one player chickens out (strategy *o*) by slowing while the other continues to speed (strategy *c*), the player who chickens out loses self-esteem (payoff of -4) and the other gains (payoff of +10). If both chicken out, their initial levels of self esteem remain unchanged (payoff +5). If neither slows, the consequences are unpleasant for both (payoff of -20 for both).

a) Represent this situation as a normal-form game.

b) Find all Nash equilibria in pure and mixed strategies.

4.4 Consider any two player game of the following form (where letters indicate arbitrary payoffs of the row player and where the payoffs of the column player are not displayed):

s_1 \ s_2	**L**	**R**
U	a, ..	b, ..
D	c, ..	d, ..

Assume a = c > b > d and display the best-response correspondence of the row player in the following graph. [Hint: Define the row player's mixed strategy to be the probability of choosing D, denote this probability by σ_{1D} and display it on the x-axis; similarly, define the column player's mixed strategy to be the probability of choosing R, denote this probability by σ_{2R} and display it on the y-axis]