

Rudolf Kerschbamer
Commitment and Information in Games

Problem Set 11

(Bayes Nash Equilibria in Infinite Static Games of Incomplete Information)

Name: _____

11.1 **(Cournot Competition with Asymmetric Information about Demand)** In a homogeneous products with inverse demand given by $P(a) = \max\{A - a, 0\}$ two firms ($i = 1, 2$) compete by simultaneously choosing quantities a_1 and a_2 (with $a = a_1 + a_2$). There are no fixed costs of production, both firms have identical constant unit costs of $c = 2$. Demand is stochastic: It is high ($A = A_H = 18$) with probability $\theta = 0.75$ and low ($A = A_L = 10$) with probability $1 - \theta$. Information is asymmetric: Firm 1 knows whether demand is high or low, firm 2 knows only the distribution of A (and the functional form of $P(a)$). Each firm's payoff is simply its profit. All of that is common knowledge.

a) Represent this situation as a (static) Bayesian game in normal form. That is, specify the set of players, the players' action spaces A_1 and A_2 , their type spaces T_1 and T_2 , their beliefs $p_1(\cdot | t_1)$ and $p_2(\cdot | t_2)$, and their payoff functions $u_1(a_1, a_2; t_1, t_2)$ and $u_2(a_1, a_2; t_1, t_2)$.

b) Specify the strategy sets of the two firms.

c) Determine the best response correspondences of both firms.

d) Solve for all Bayes Nash equilibria in pure strategies.

11.2 **(Cournot Competition with Asymmetric Information about Unit Costs)** In a homogeneous products market with inverse demand given by $P(a) = \max\{1 - a, 0\}$ two firms ($i = 1, 2$) compete by simultaneously choosing quantities a_1 and a_2 (where $a = a_1 + a_2$). The constant unit costs of each firm are high ($c_i = c_H = 0.4$) with $\theta = 0.4$ and low ($c_i = c_N = 0.2$) with probability $1 - \theta$. The realisation of c_i is independent of the realisation of c_j . Each firm knows its own unit costs but not those of its competitor. Each firm's payoff is simply its profit. All of that is common knowledge.

a) Represent this situation as a (static) Bayesian game in normal form. That is, specify the set of players, the players' action spaces A_1 and A_2 , their type spaces T_1 and T_2 , their beliefs $p_1(\cdot | t_1)$ and $p_2(\cdot | t_2)$, and their payoff functions $u_1(a_1, a_2; t_1, t_2)$ and $u_2(a_1, a_2; t_1, t_2)$.

b) Specify the strategy sets of the two firms.

c) Determine the best response correspondences of both firms.

d) Solve for all Bayes Nash equilibria in pure strategies.