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Commitment and Information in Games

Problem Set 10

(Bayes Nash Equilibria in Finite Static Games of Incomplete Information) (Plus some Exercises on Bayesian Updating)

Name: _ _ _ _ _ _ _ _ _ _ _ _

10.1 Bob (he) has fallen in love with Anne (she). While at separate workplaces, they must choose to attend either a football match (action F) or the theater (action T). Since Bob does not know Anne very well, he is not sure whether she actually wants to spent time with him. If she likes him they play the following game of “Battle of the Sexes” (he is the row player, she is the column player):

	F	**T**
F	3, 1	0, 0
T	0, 0	1, 3

If she does not like him they play the following game (again he is the row player, she the is column player):

	F	**T**
F	3, 0	0, 1
T	0, 3	1, 0

Bob believes that Anne is in love with him with probability $\mu = 1/3$ (i.e., that they are playing the first game with probability *1/3*). Anna knows wheter she loves Bob or not.

a) Formulate this strategic decision situation as a Baysian game in normal-form.

b) Draw the extensive form of the Bayesian game.

c) Derive the (pure strategy Bayesian) best response correspondences of both players.

d) Solve fore all Bayes Nash equilibria in pure strategies.

e) Assume now that Bob believes that Anne likes him with probability $\mu = 4/5$. Solve for all Bayes Nash equilibria in pure strategies.

10.2 Consider the following strategic decision situation. Two opposed armies are poised to seize an island. Each army's general can Choose either "attack" or "not attack". In addition, each army is either "strong" or "weak" with equal probability where the draws for each army are independent and an army's type is known only to its general. Payoffs are as follows: The island is worth 100 if captured. An army can capture the island either by attacking when its opponent does not, or by attacking when its rival does if it is strong and iis rival isweak. If two armies of equal strenght both attack, neither captures the island. An army also has a cost of feighting, which is 40 if it is strong and 60 if it is weak. There is no cost of attacking if its rival does not.

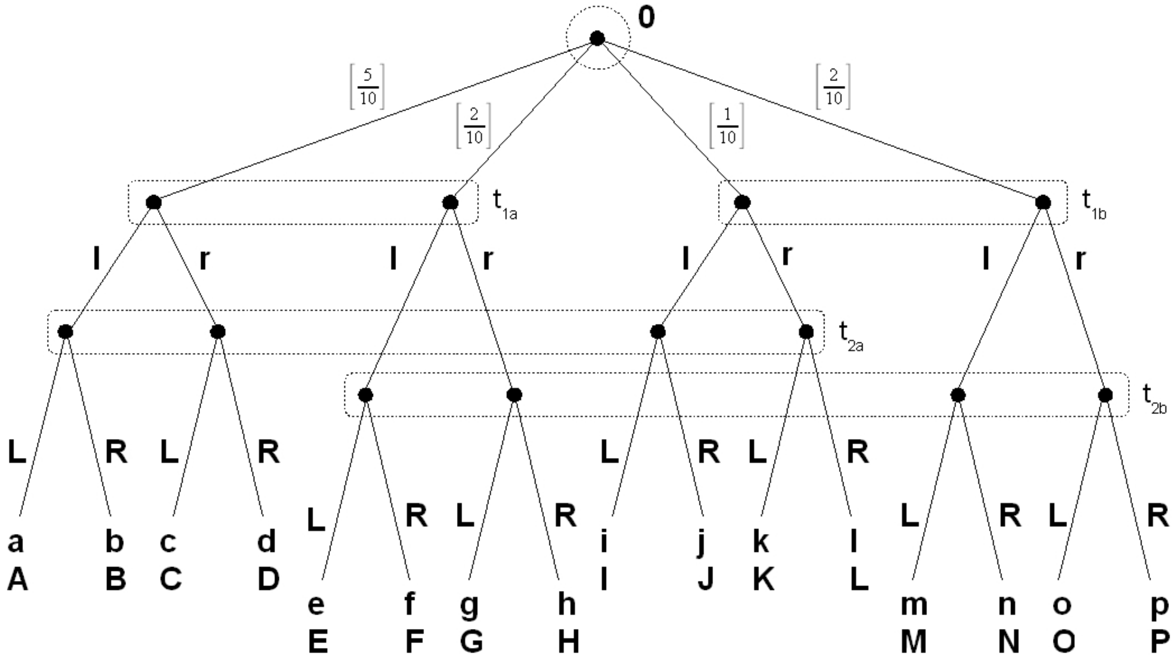
a) Represent this strategic decision situation as a Baysian game in normal form.

b) Draw the extensive form of the Baysian game.

c) Derive the (pure strategy, Bayesian) best response correspondences of the two armies.

d) Solve for all pure strategy Bayesian Nash equilibria of this game.

10.3 Consider the Baysian game in extensive form below. Find conditions for the payoffs (a,b,c,...and A,B,C,...) such that the strategy combination $(s_1,s_2) = ((s_1(t_{1a}), s_1(t_{1b})), (s_2(t_{2a}), s_2(t_{2b}))) = ((r, l), (L, R)$ forms a (pure strategy) Bayes Nash equilibrium.



10.4 **(Bayes Rule 1)** A company can invest into a property that contains valuable oil reserves with probability $p = ½$, and only dead rock with probability $1 - p$. There is a possibility to get more information by performing a test drilling. There are two possible outcomes of the drilling: "successful" (it is likely that there are oil reserves) or "unsuccessful" (it is not very likely that there are oil reserves). Yet even a successful drilling is no guarantee that there is an exploitable oil field: if there is an exploitable oil field, the drilling is successful with *60%* probability; if there is no exploitable oil field, there is still a chance of *20%* of a successful drill.

a) Assume the drilling has been successful. What is the probability that there is an exploitabile oil field?

b) Assume the drilling has not been successful. What is the probability that there is an exploitabile oil field?

10.5 **(Bayes Rule 2)** The probability to get a smurf (S) when opening a surprise egg is $Pr(S) = 1/7$, the probability to get no smurf, i.e. the probability that the egg is empty (E), is $Pr(E) = 6/7$. To get a clue whether the egg contains a smurf one can apply a shaking test. A shaking test has two possible outcomes: "positiv" (P; i.e. the egg most likely contains a smurf) and "negative" (N; i.e. the egg most likely does not contain a smurf). The probability that an egg contains a smurf which is not detected by the shaking test is $Pr(N \mid S) = 30\%$. The probability that the shaking test predicts a smurf mistakenly is $Pr(P \mid E) = 20\%$.

a) Assume the shaking test was positive. What is the probability that the tested egg is indeed inhabited by a smurf?

b) Assume the shaking test was negative. What is the probability that the tested egg is nevertheless inhabited by a smurf?