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Commitment and Information in Games

Problem Set 1

(Representation of Games)

Name: _ _ _ _ _ _ _ _ _ _ _

1.1 **(Battle of the Sexes)** Oscar and Tina meet accidentally at a cafe. They fall in love at first sight and chit-chat animatedly. Coming up with their plans for the evening, Tina suggests spending the evening together at the football stadium. But Oscar is a cinema fan and suggests watching the latest Woody Allen movie together. During the discussion about the place to meet in the evening he suddenly realizes that he has almost forgotten an important job interview. He leaves in a hurry without fixing the rendezvous place and without exchanging mobile phone numbers.

a) Represent this situation as a normal-form and as an extensive-form game. Assume that there are only two possible rendezvous points: the football stadium and the cinema. When specifying payoffs bear in mind that Tina prefers the football match whereas Oscar prefers the movie; if they miss each other, however, both would lose any pleasure from their favourite entertainment.

b) Specify the formal elements of the extensive-form representation of this game (i.e., specify the set of players, the set of histories, the set of terminal histories, the player function, the information partition of each player, and the payoff function of each player).

1.2 **(Prisoners' Dilemma)** Two suspects are arrested and charged with a crime. They are put into separate cells and are not allowed to talk to each other. The attorney is sure that they are responsible for a major crime but he lacks sufficient evidence to convict them, unless at least one confesses. So he goes to each of them and explains him the consequences that will follow from the actions they can take. If neither confesses then both will be convicted of illegal gun possession and sentenced to three months in jail. If both of them confess, then both will get a more severe punishment but not the maximum sentence. Finally, if only one of them confesses but the other does not, then the confessor will get away as chief witness, while his companion will face the maximum sentence.

Represent this situation as a normal-form and as an extensive-form game. In the representation normalize the payoffs when both suspects confess to zero and assume that the suspects want to minimize their time behind bars.

1.3 **(Market Entry)** A monopoly (the “incumbent”) earns a profit of 100 million Euro. A potential competitor (the “challenger”) considers entering the market. If he does not enter his profit is zero. If he enters, the incumbent has to decide whether to fight aggressively, whereas both competitors suffer a loss of € 10m, or not. In the latter case both firms get a duopoly profit of € 40m.

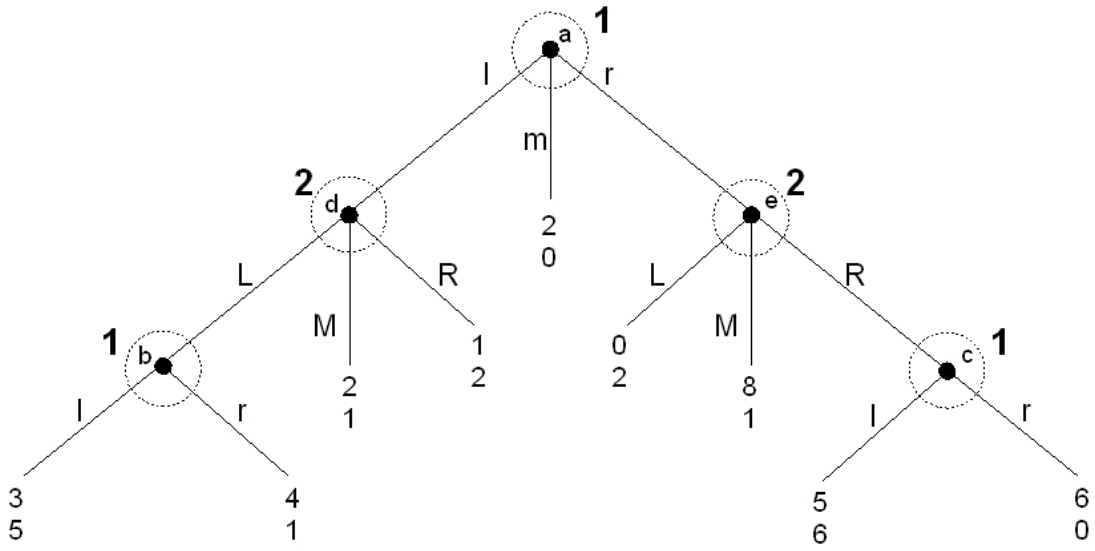
a) Represent this situation as a normal-form and as an extensive-form game.

b) Specify the formal elements of the extensive-form representation of this game (i.e., specify the set of players, the set of histories, the set of terminal histories, the player function, the information partition of each player, and the payoff function of each player).

c) What is a strategy in the extensive-form representation of this game? Specify the pure strategy sets of the two players.

1.4 **(Cournot Competition)** In a homogeneous product market with inverse demand given by $P(x) = max\ \{1-x\ ,\ 0\}$ two firms $(i = 1,2)$ compete by simultaneously choosing output quantities s_1 and s_2 (where $x = s_1 + s_2$). Both have the same cost function $C(s_i) = 0.1\ s_i$. Assume that the competitors can choose any quantity between 0 and 1 ($s_1, s_2 \in [0,1]$) and represent this situation as a normal-form game.

1.5 Consider the extensive-form game below.



a) List all pure strategies of players 1 and 2.

b) List all histories and all terminal histories

c) Which strategy profiles (s_1, s_2) lead to the terminal history rRl?

d) Transform the extensive-form game into its normal-form representation.

1.6 In an extensive-form game where player i has X information sets indexed $x = 1, ..., X$ and Y_x possible actions at information set x, how many pure strategies does player i have? [Allow for $Y_x \neq Y_{x'}$ for $x \neq x'$!]

1.7 Consider the following normal-form game:

s_2 / s_1	**L**	**R**
O	4, -2	-2, 2
U	10, 11	-3, 6

Suppose the players mix according to $\sigma = (\sigma_1,\sigma_2) = ((\sigma_{1O}, \sigma_{1U}), (\sigma_{2L}, \sigma_{2R})) = ((3/5,2/5),(¼ , ¾))$. Calculate $U_1(\sigma)$ and $U_2(\sigma)$. [Hint.: Don't try to justify the players' behaviour - this is just an exercise to calculate expected utilities associated with mixed strategy profiles.]

1.8 **(Tom and Jerry)** Jerry (she) can hide in the kitchen, in the bathroom or in the bedroom. Tom (he) can look for her at only one of these places. If he searches at the right place, he finds her and wins. If he searches at the wrong place, he loses and Jerry wins.

a) Assume that Tom can observe where Jerry hides before he starts searching. Represent this situation as a normal-form game (bi-matrix) and as an extensive-form game (game tree). Also, specify the formal elements of the extensive-form representation of this game (i.e., specify the set of players, the set of histories, the set of terminal histories, the player function, the information partition of each player, and the payoff function of each player).

b) Assume now that Jerry knows where Tom will look for her before she hides. Represent this situation as a normal-form game (bi-matrix) and as an extensive-form game (game tree). Also, specify the formal elements of the extensive-form representation of this game (i.e., specify the set of players, the set of histories, the set of terminal histories, the player function, the information partition of each player, and the payoff function of each player).

c) Represent the situation where neither Tom nor Jerry can observe what his/her partner does as a normal-form game (bi-matrix) and as an extensive-form game (game tree). Also, specify the formal elements of the extensive-form representation of this game (i.e., specify the set of players, the set of histories, the set of terminal histories, the player function, the information partition of each player, and the payoff function of each player).